Body Wave Tomography

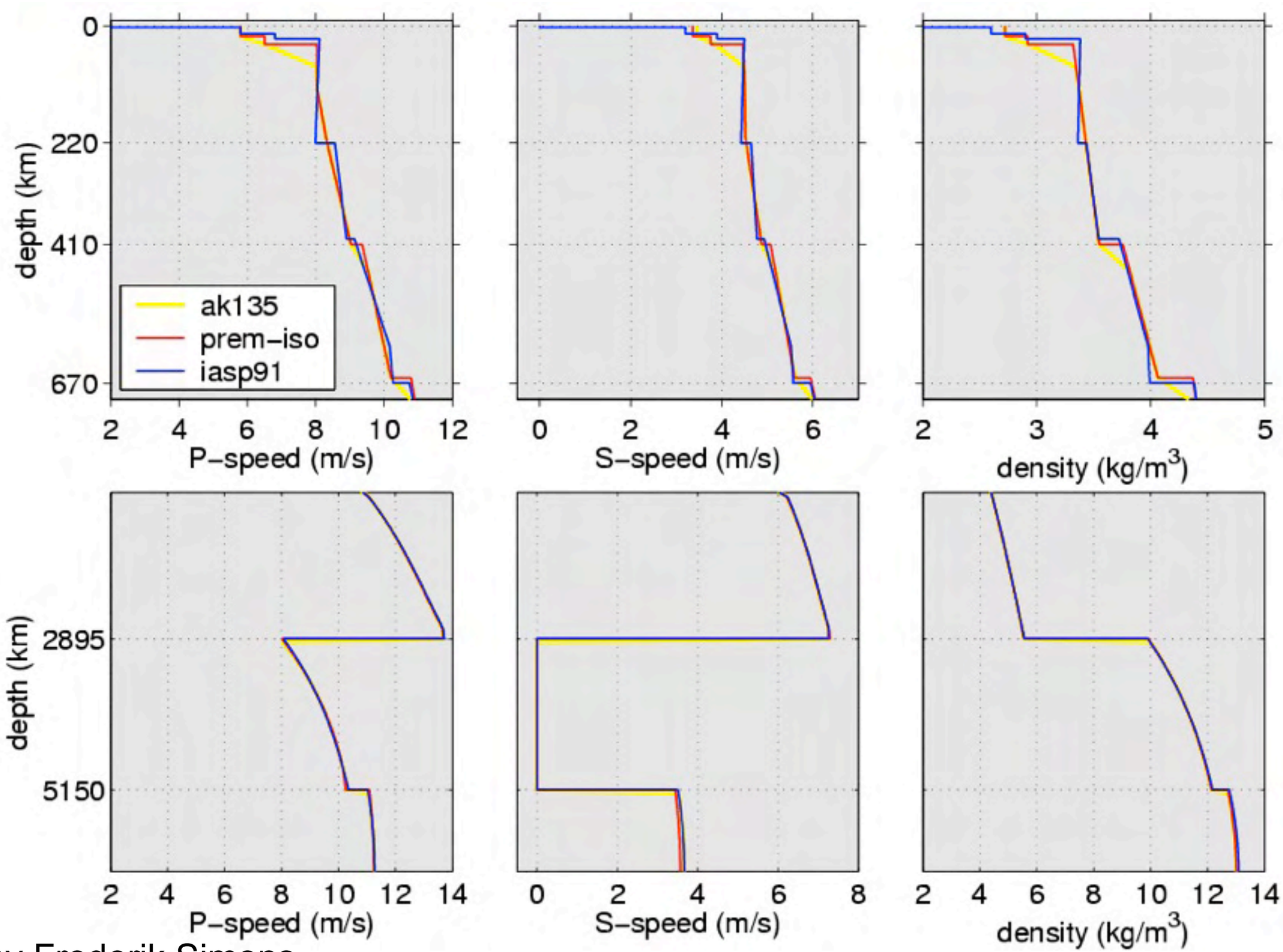
Matt Fouch

ASU EarthScope Seminar
January 26, 2010

Seismic Tomography Mini-FAQ

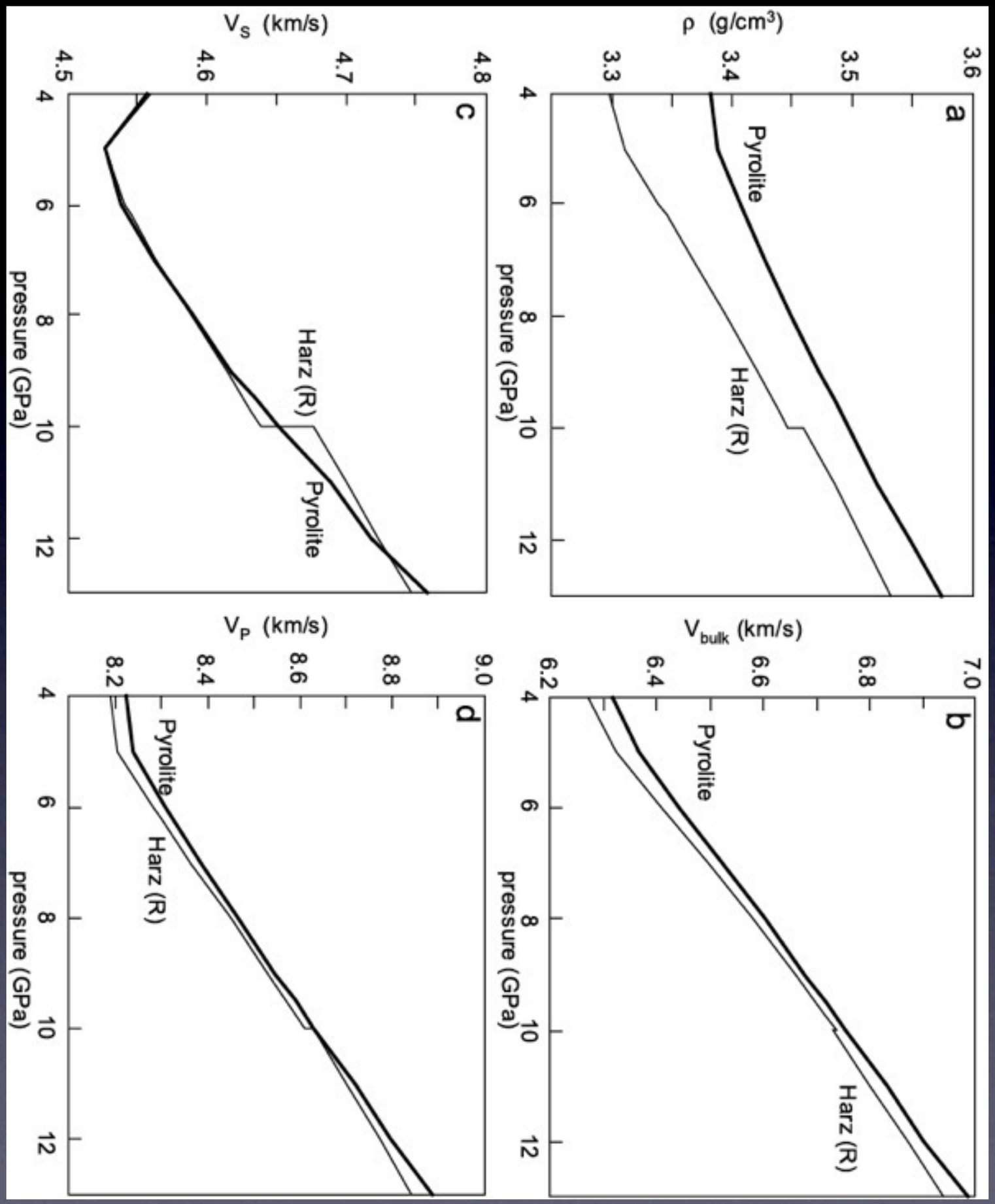
- *What is it?*
  - Earth sciences “CAT-scan”
  - Measure of seismic wavespeeds or wavespeed variations
- *Why do we care?*
  - Understand phase/temperature/compositional variations
  - Relate geology and mantle dynamics to regional processes

The reference Earth: Radial models



Courtesy Frederik Simons

Seismic Velocity Relationships



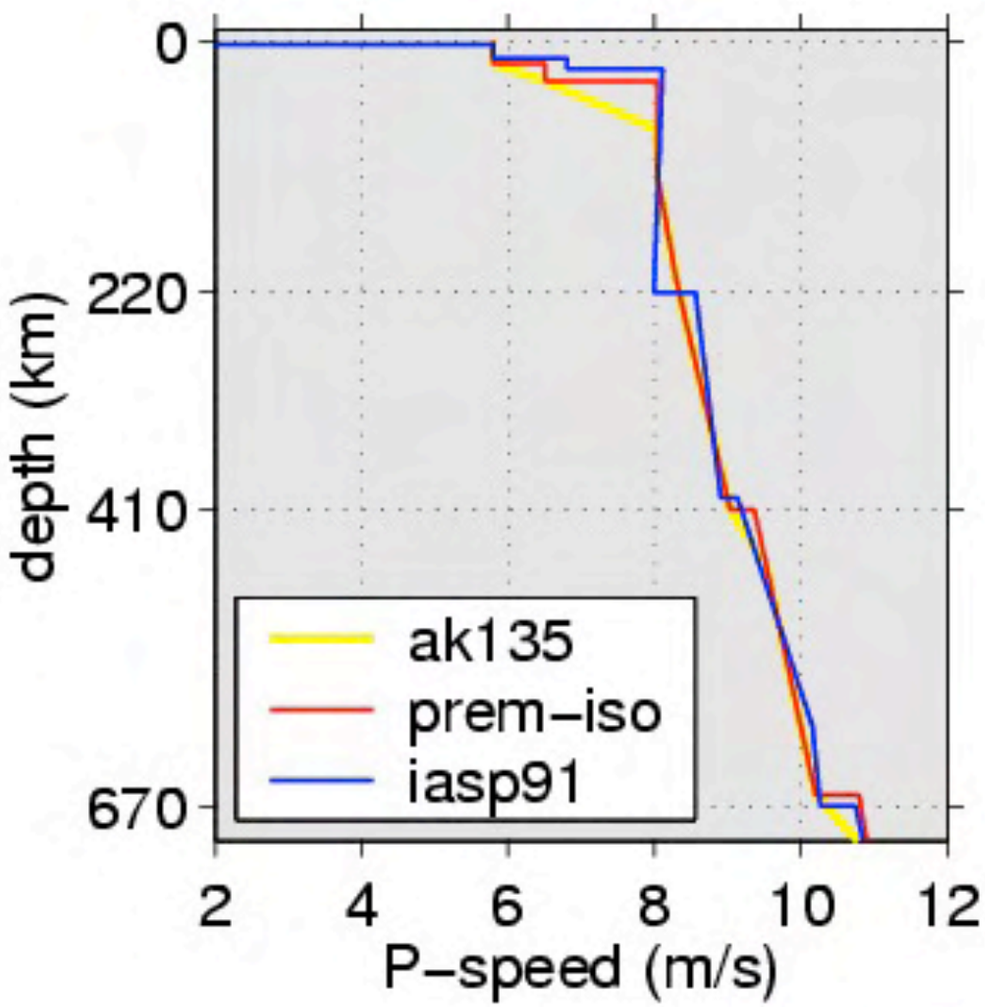
- Determined by
  - Pressure (depth)
  - Temperature
  - Composition

Matsukage et al. [2005]

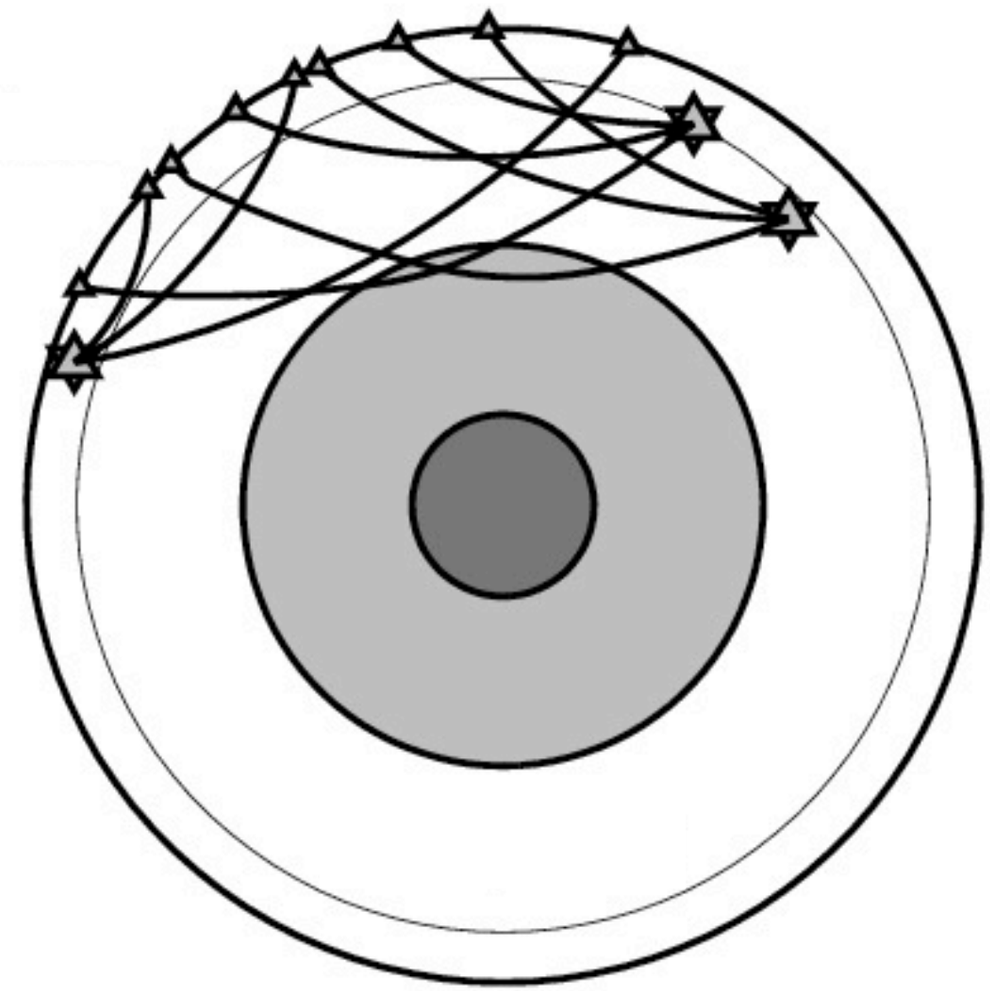
Courtesy Frederik Simons

Forward modeling of the wave field, Part I:

Ray tracing, most 1-D



Before



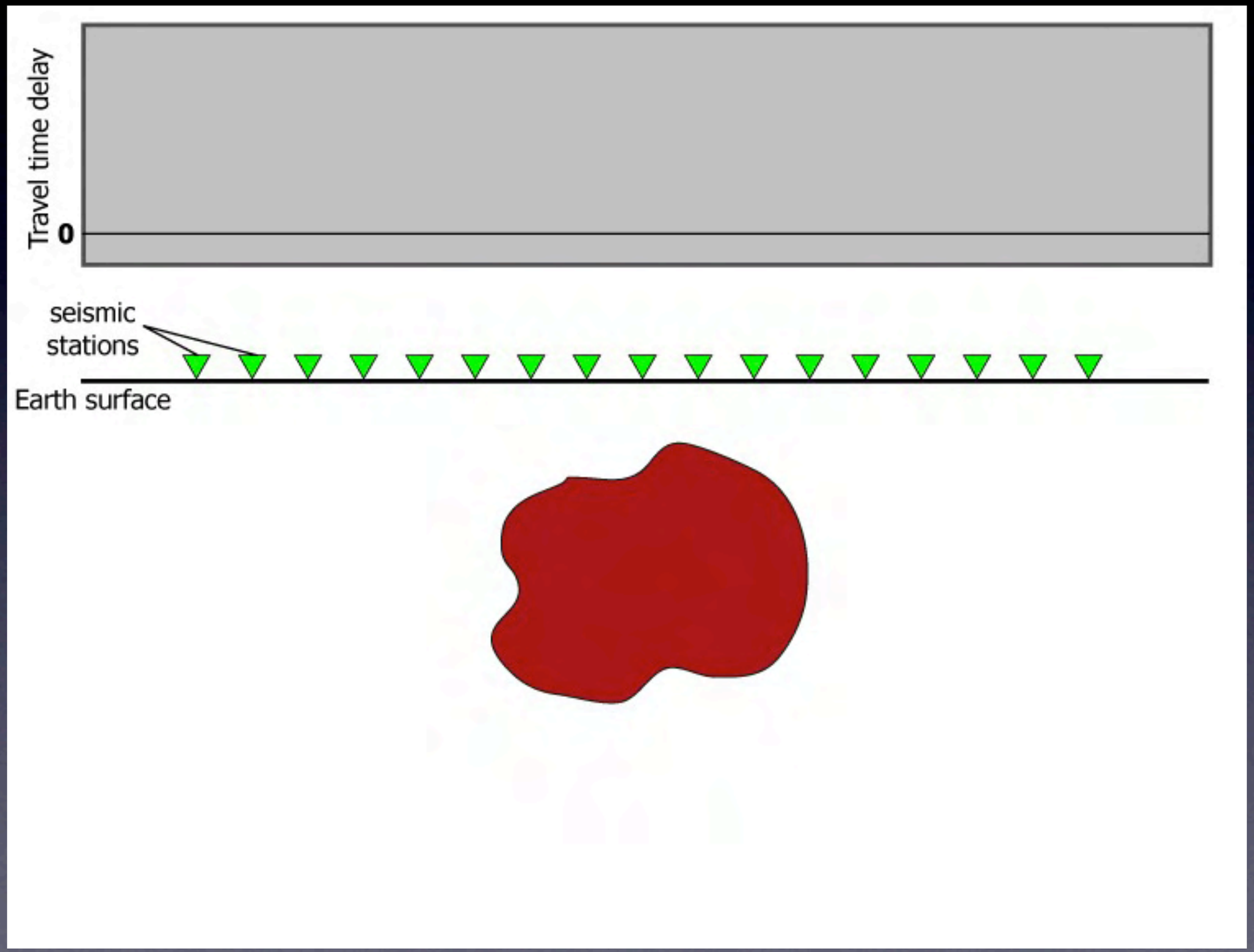
After

Kennett, *GJI*, 1995

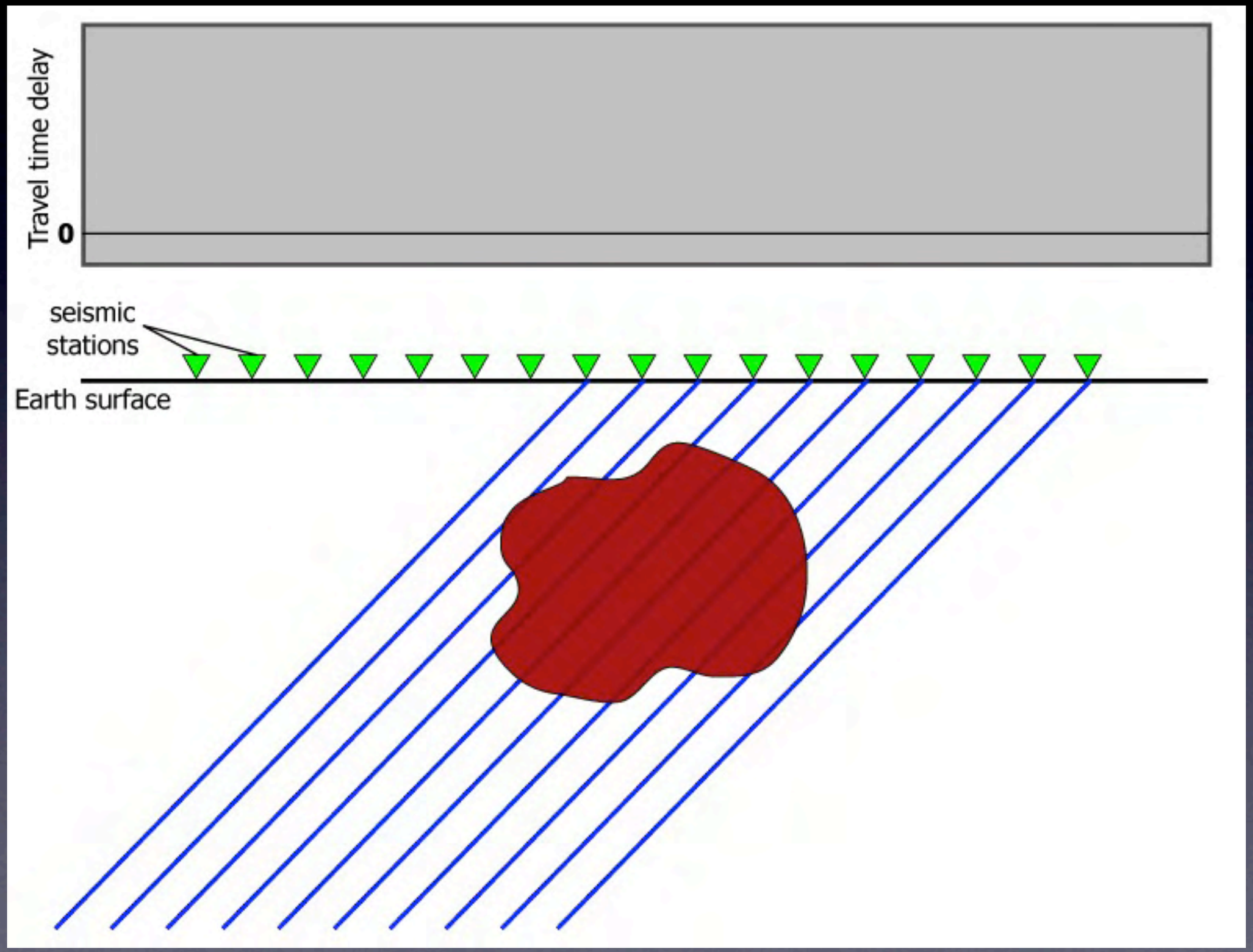
Bullen & Bolt, 1985

Buland, *BSSA*, 1983

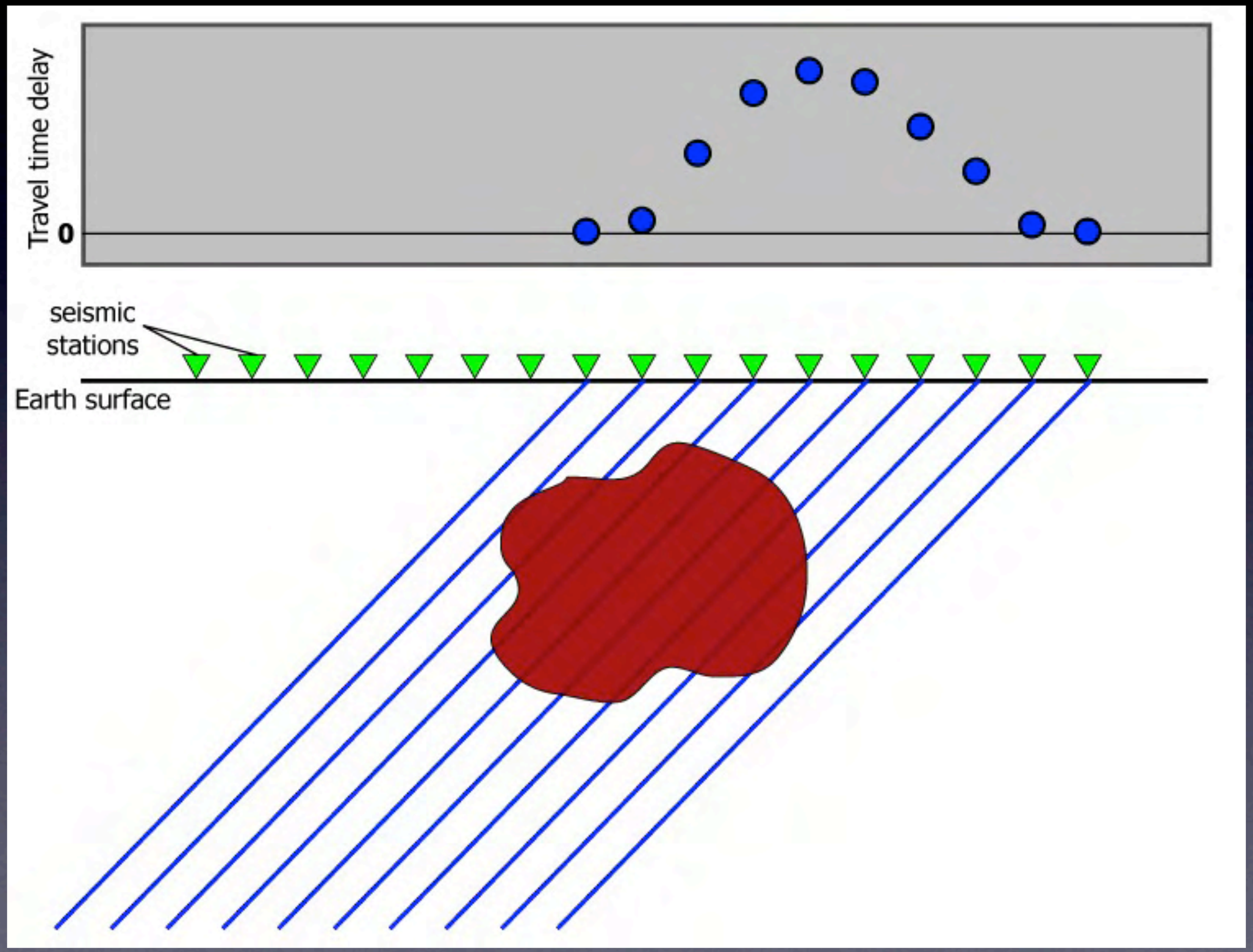
Relative Delay Time Example



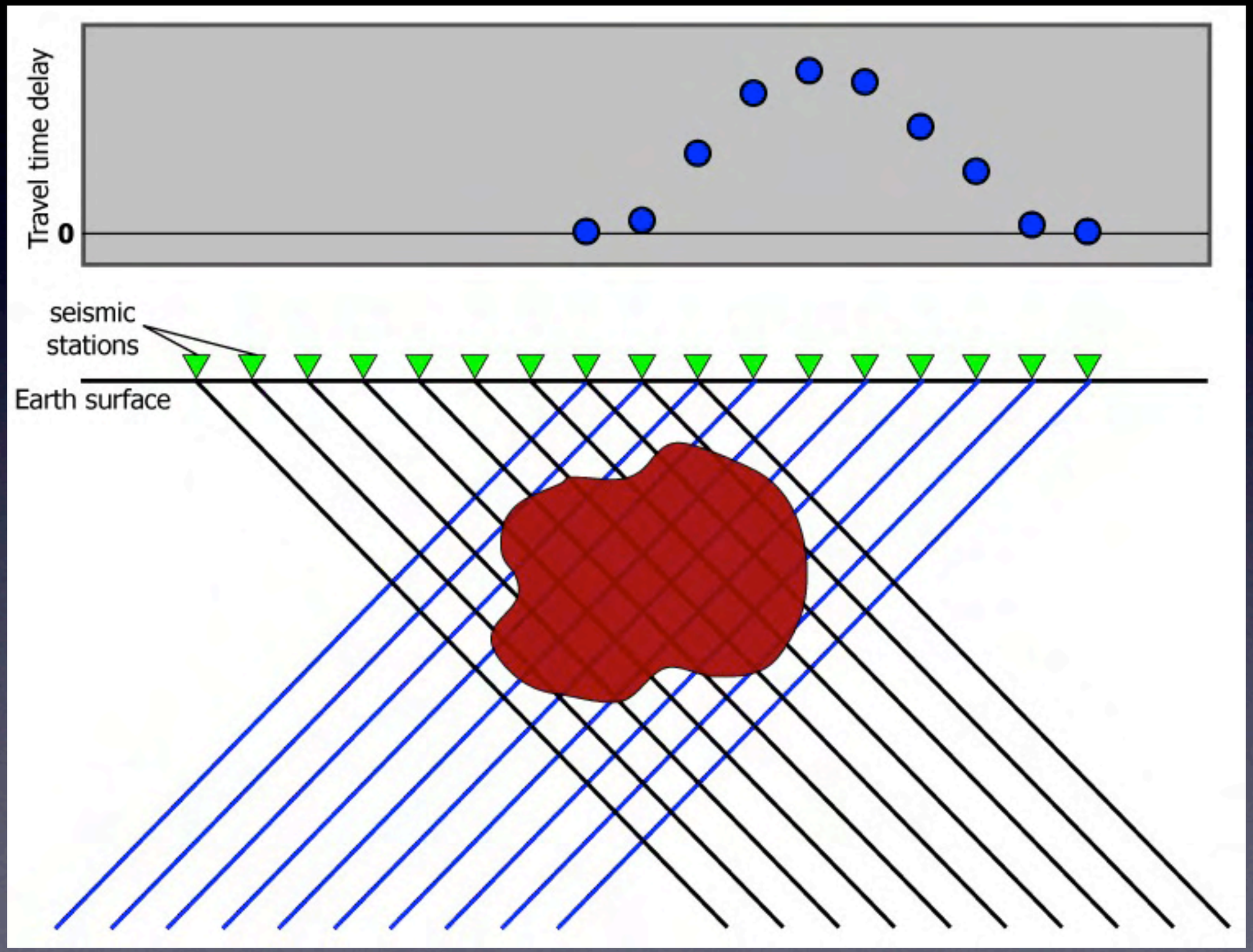
Relative Delay Time Example



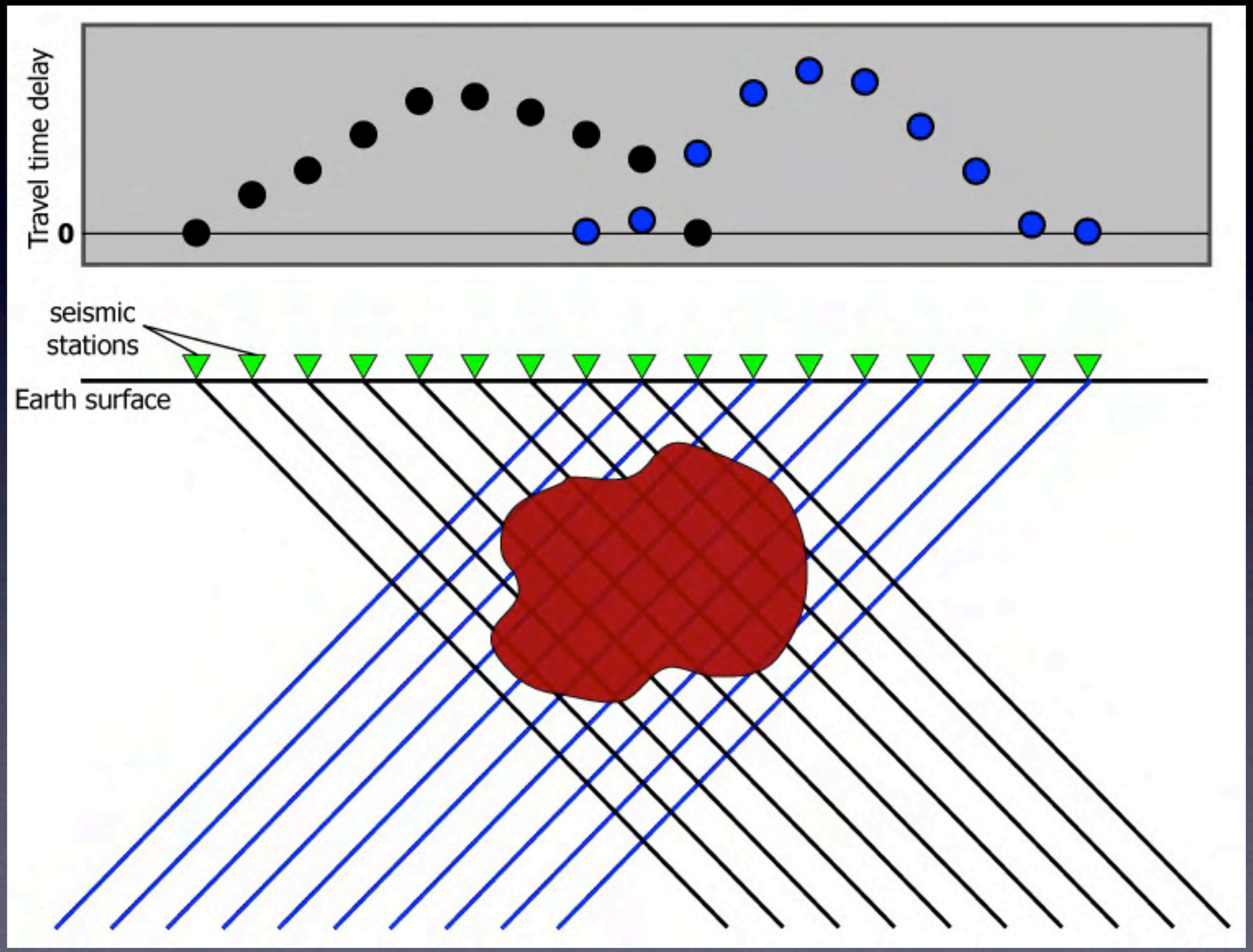
Relative Delay Time Example



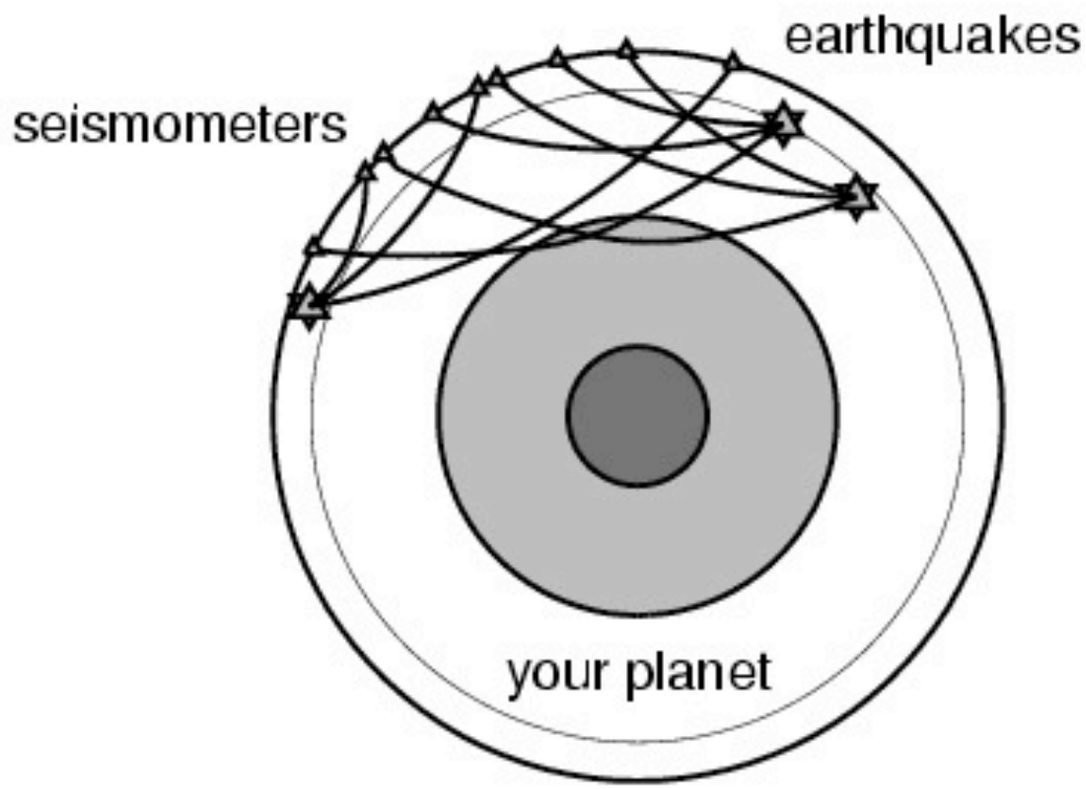
Relative Delay Time Example



Relative Delay Time Example



What is $f(x, y, z)$? Seismic wavespeeds.



Travel-time tomography

The Earth is made of a heterogeneity of seismic velocities $v(x, y, z)$.

Travel-time anomalies go as

$$\delta t = \int_{\text{ray}} \frac{1}{\delta v(x, y, z)} \, ds. \qquad (3)$$

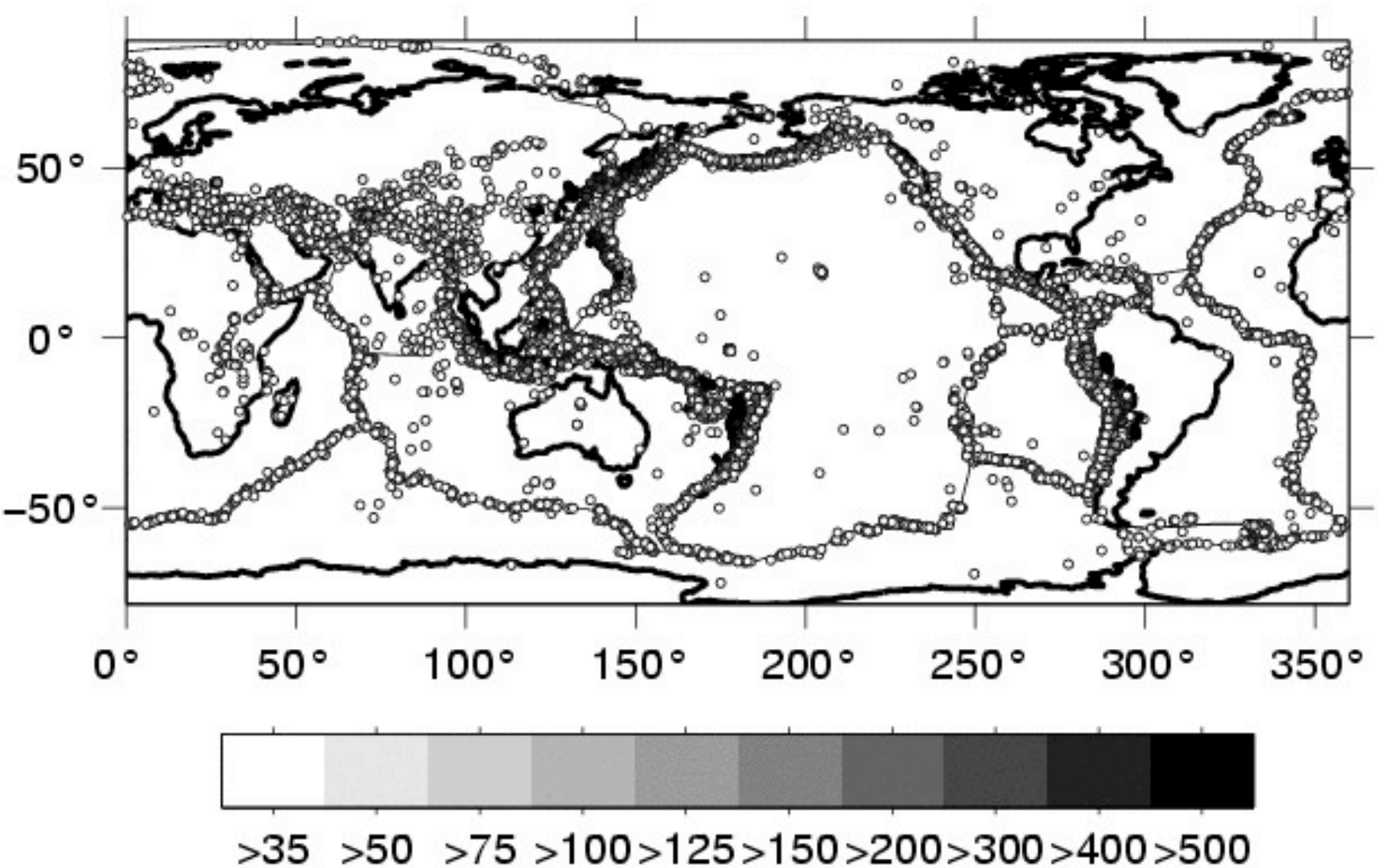
Waveform tomography

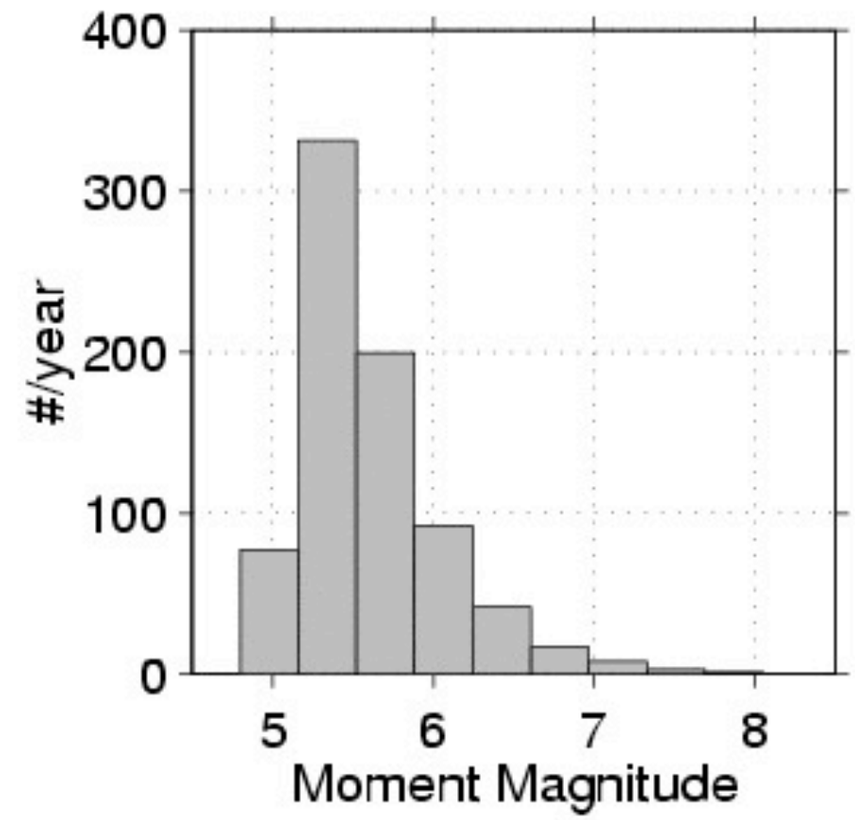
Arrival times depend on the wavelength of the seismic phases.

All raypaths curve and coverage is far from perfect.

Courtesy Frederik Simons

Non-continuous source coverage



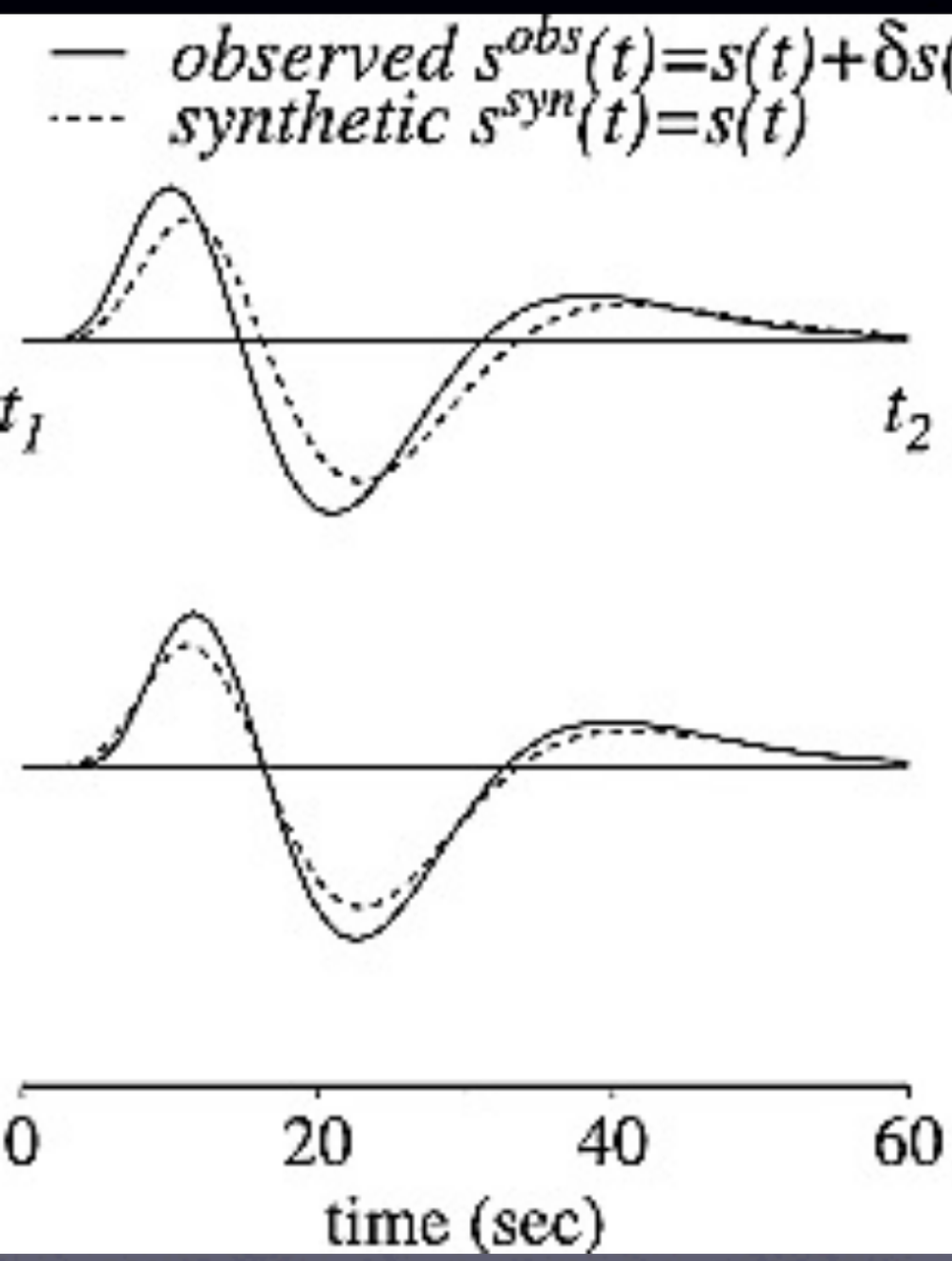


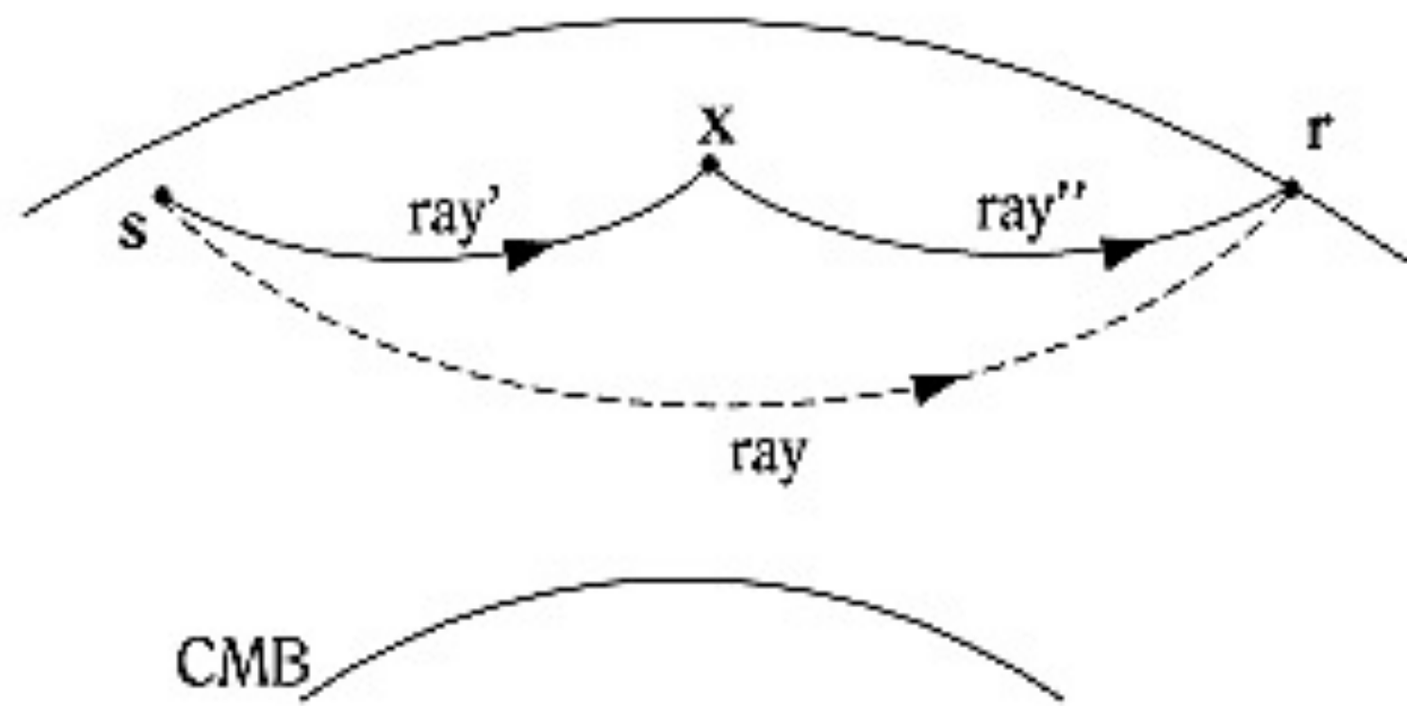
The CMT catalog of large events

Courtesy Frederik Simons

Ray Theory Paths

- Spaghetti, bananas, and doughnuts

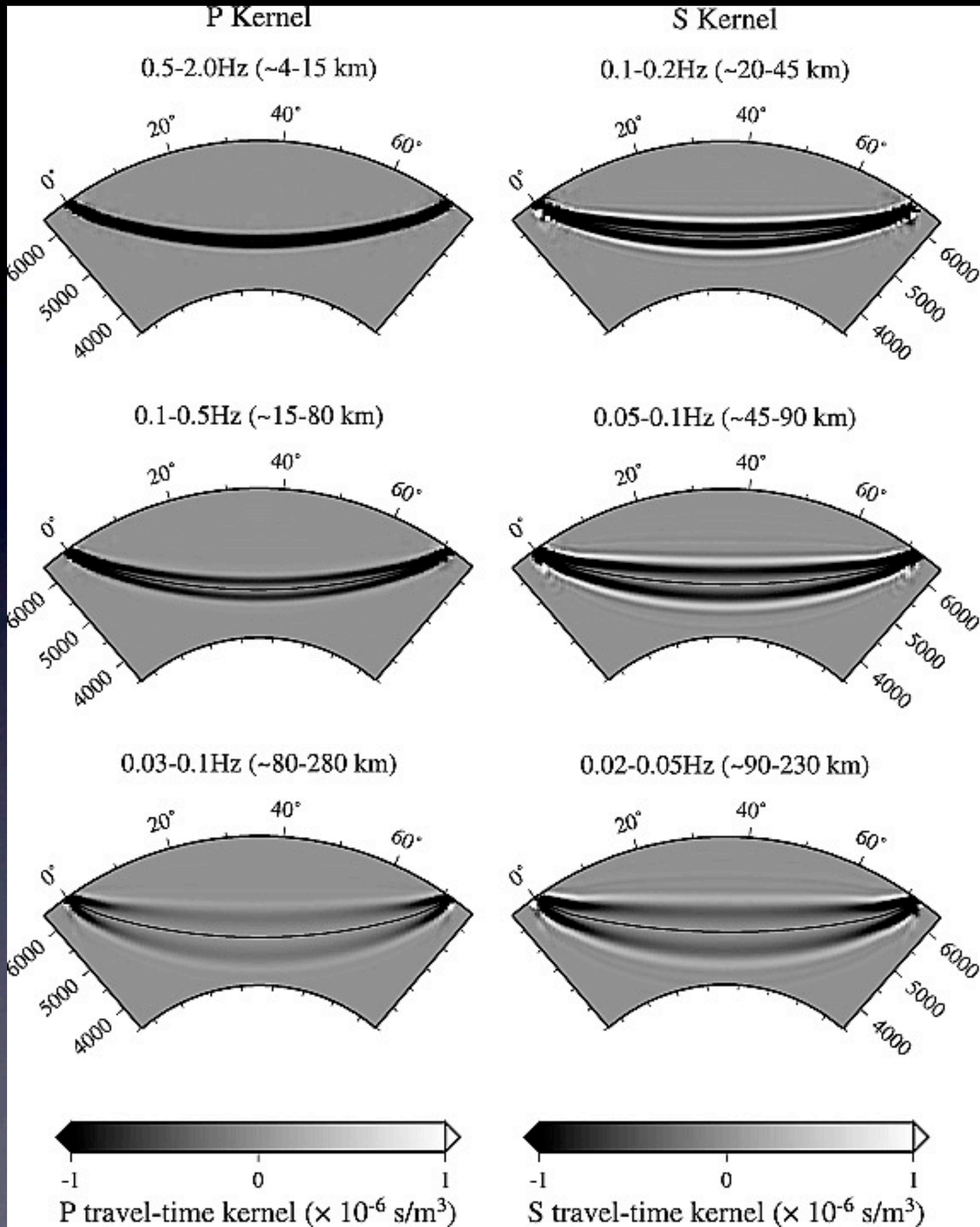




Hung et al. [2004]

Finite Frequency Paths

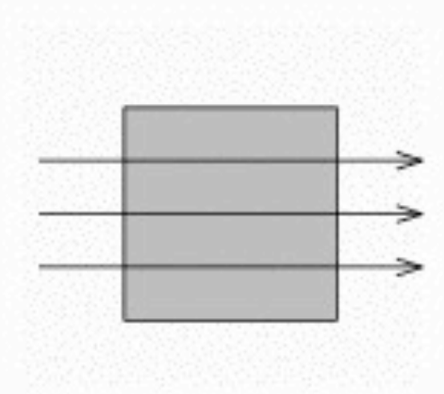
- Spaghetti, bananas, and doughnuts



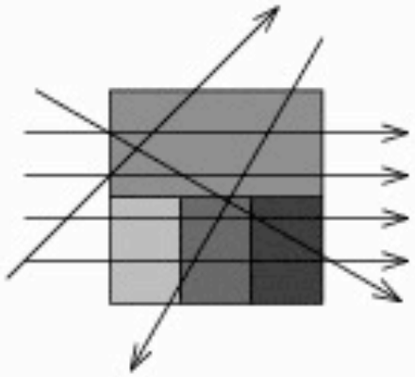
Hung et al. [2004]

Letting it simmer: Solving inverse problems

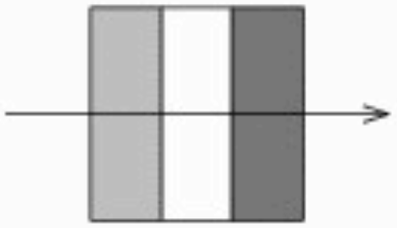
We have:	$\mathbf{G} \cdot \mathbf{m} = \mathbf{d}$,	which is **linear**.
You think:	$\mathbf{m} = \mathbf{G}^{-1} \cdot \mathbf{d}$,	but we **can't invert** a non-square $M \times N$ matrix.
You think:	$\mathbf{G}^{\mathrm{T}} \cdot \mathbf{G}$	is square, let's solve $\mathbf{G}^{\mathrm{T}} \cdot \mathbf{G} \cdot \mathbf{m} = \mathbf{G}^{\mathrm{T}} \cdot \mathbf{d}$.
You try:	$\mathbf{m} = (\mathbf{G}^{\mathrm{T}} \cdot \mathbf{G})^{-1} \cdot \mathbf{G}^{\mathrm{T}} \cdot \mathbf{d}$.	
Alas!	$\mathbf{G}^{\mathrm{T}} \cdot \mathbf{G}$	may be singular, ill-conditioned, under/over-determined, have (near-)zero eigenvalues, and thus be **not-invertible**.



over-determined, M>N



mixed-determined



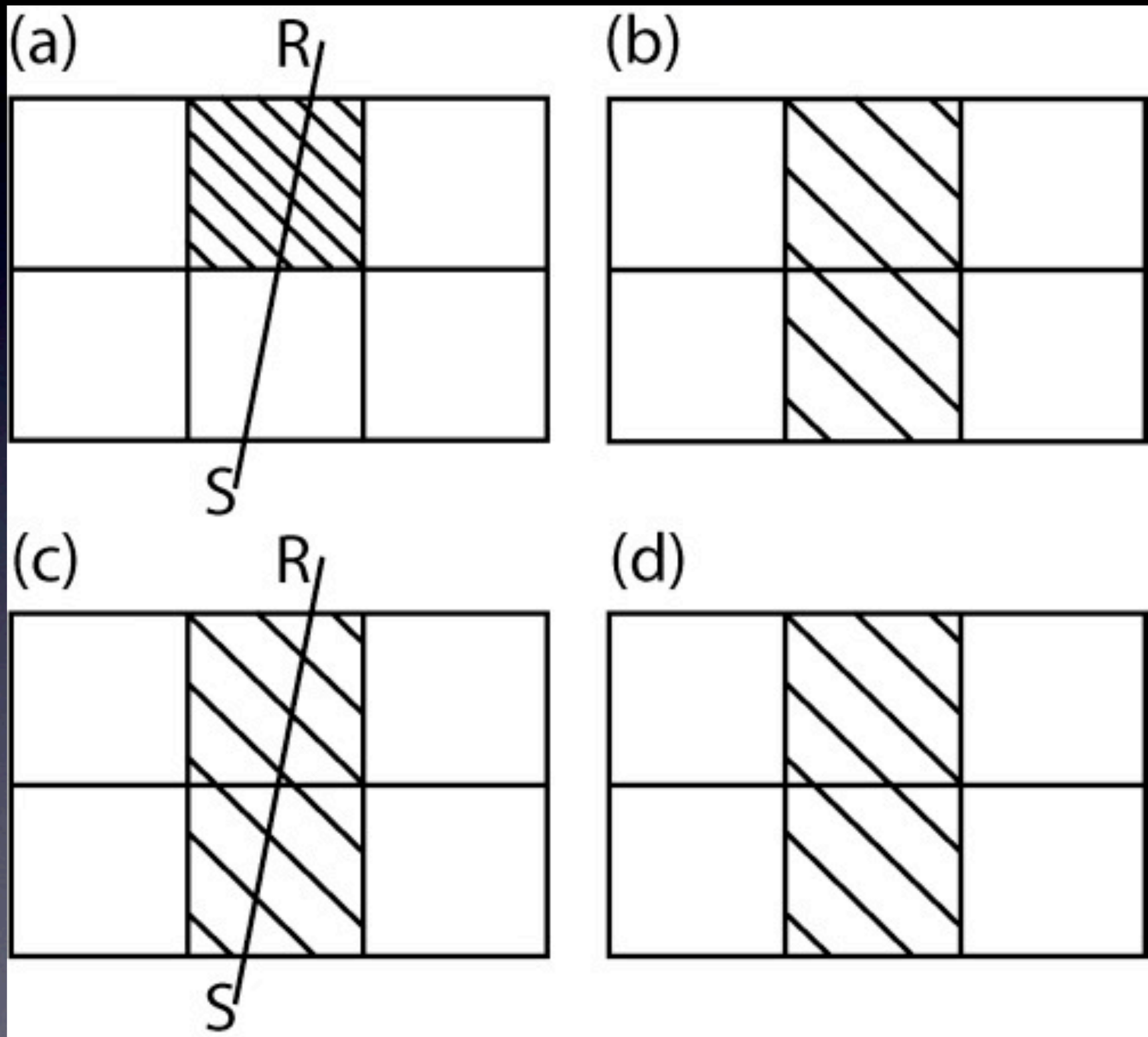
under-determined, M<N

Courtesy Frederik Simons

Menke, 1989

Resolution issues

- Different models, same results (underdetermined)
- Need crossing rays (unique information)



Allen and Tromp [2005]

Body Wave Tomography: Strengths

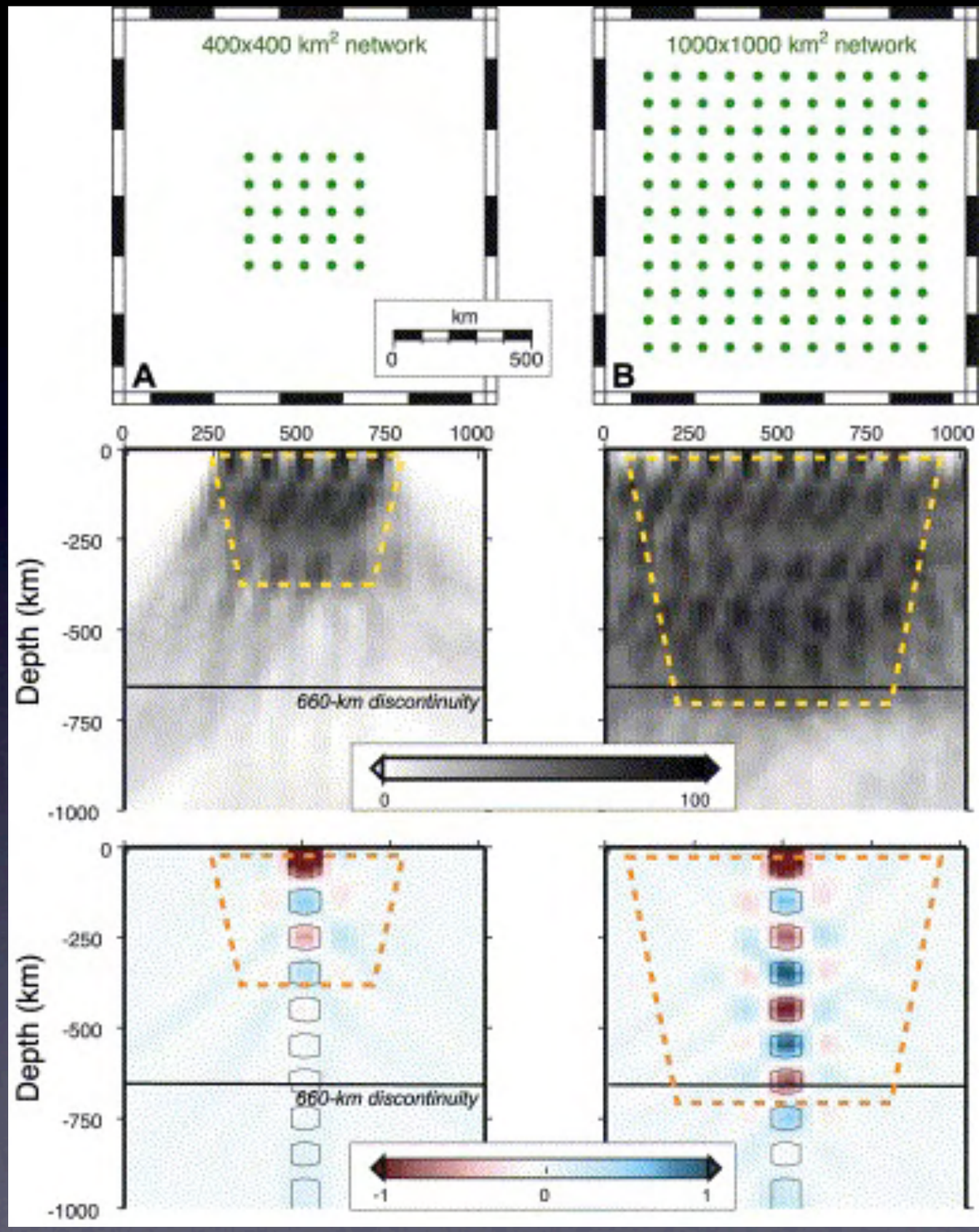
- Many methods: great when they agree!
- Results can be easy to interpret visually
- Dense arrays provide higher level of resolution
- Surface wave inversions can provide absolute velocities
- Complementary observations:
  - Receiver functions
  - Seismic anisotropy
  - Combined surface wave/body wave inversions

Body Wave Tomography: Weaknesses / Caveats

- Resolution: Model will always be a blurry (smoothed) image of real structure
- Regularization: How much bias does model have from damping and smoothing?
- Misfit: How well does model fit data? Is it unique?
- Many methods: What happens when they don't agree?
- Be cognizant of:
  - Very small features: avoid interpretation!
  - Color schemes and isocontours
  - Streaks pointing in direction of earthquakes used in inversion

Model Sampling

- Array aperture and raypath density are very important



Ritsema and Allen [2003]

Uncertainties and Resolution

- Checkerboard test
- Spike test (Resolution matrix)
- Statistical test (bootstrapping, Jack-knifing methods)
- Synthetic test (simple model checking)
- Probabilistic Bayesian approach (Tarantola & Valette, 1982)
- Covariance matrices (model and data space)

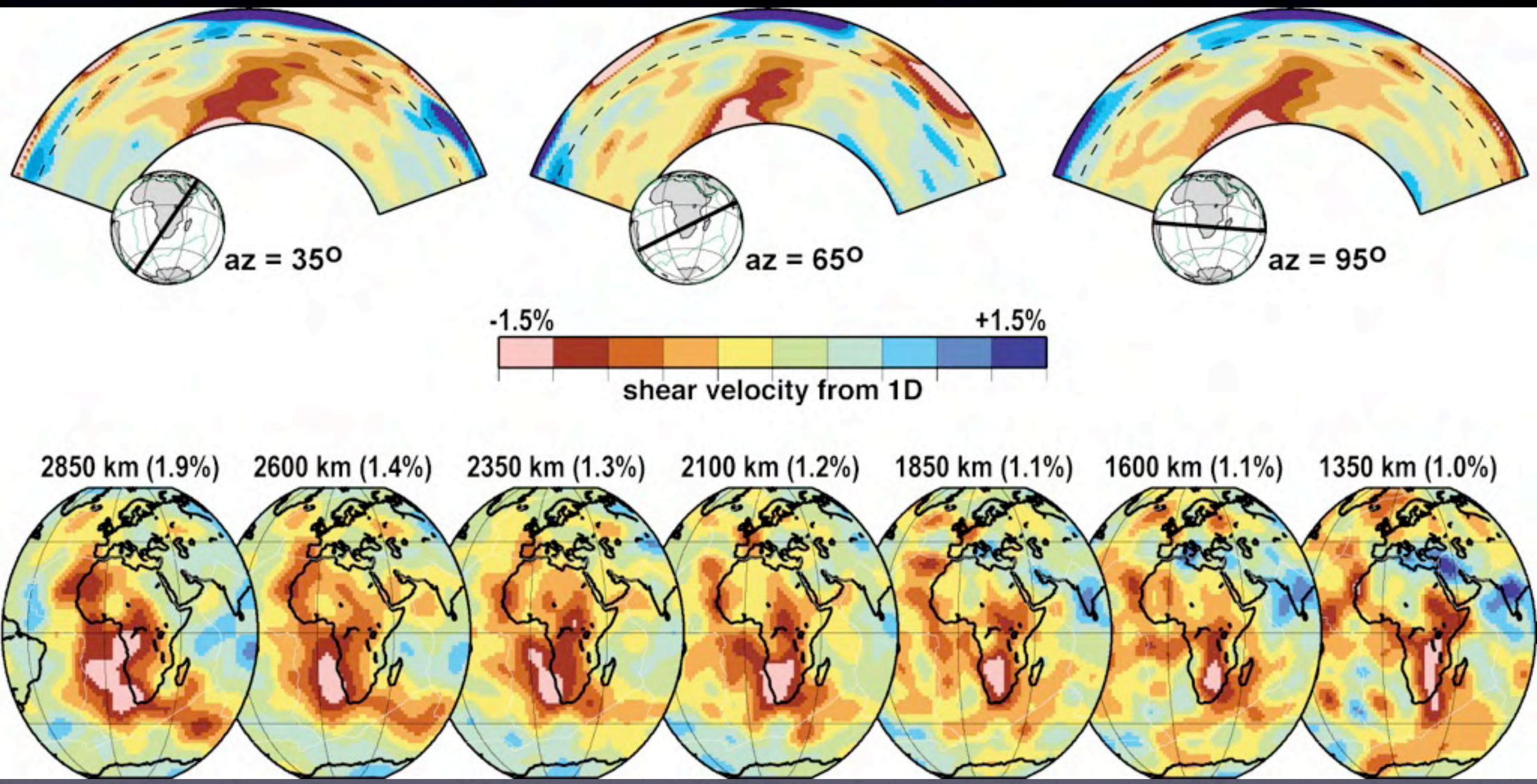
From http://www-lgit.obs.ujf-grenoble.fr/~virieuxj

TOMOGRAPHY TARGETS

- Local scale (20 km x 20 km x 10 km)
  - characteristic DH ~ 4 km and DZ ~1 km
- Lithospheric scale (100 x 100 x 50 km)
  - DH ~ 15 km and DZ ~ 5 km
- Mantle scale (3000 x 3000 x 600 km)
  - DH ~ 50 km and DZ ~ 50 km
- Global scale (Whole Earth) D ~ 500 km

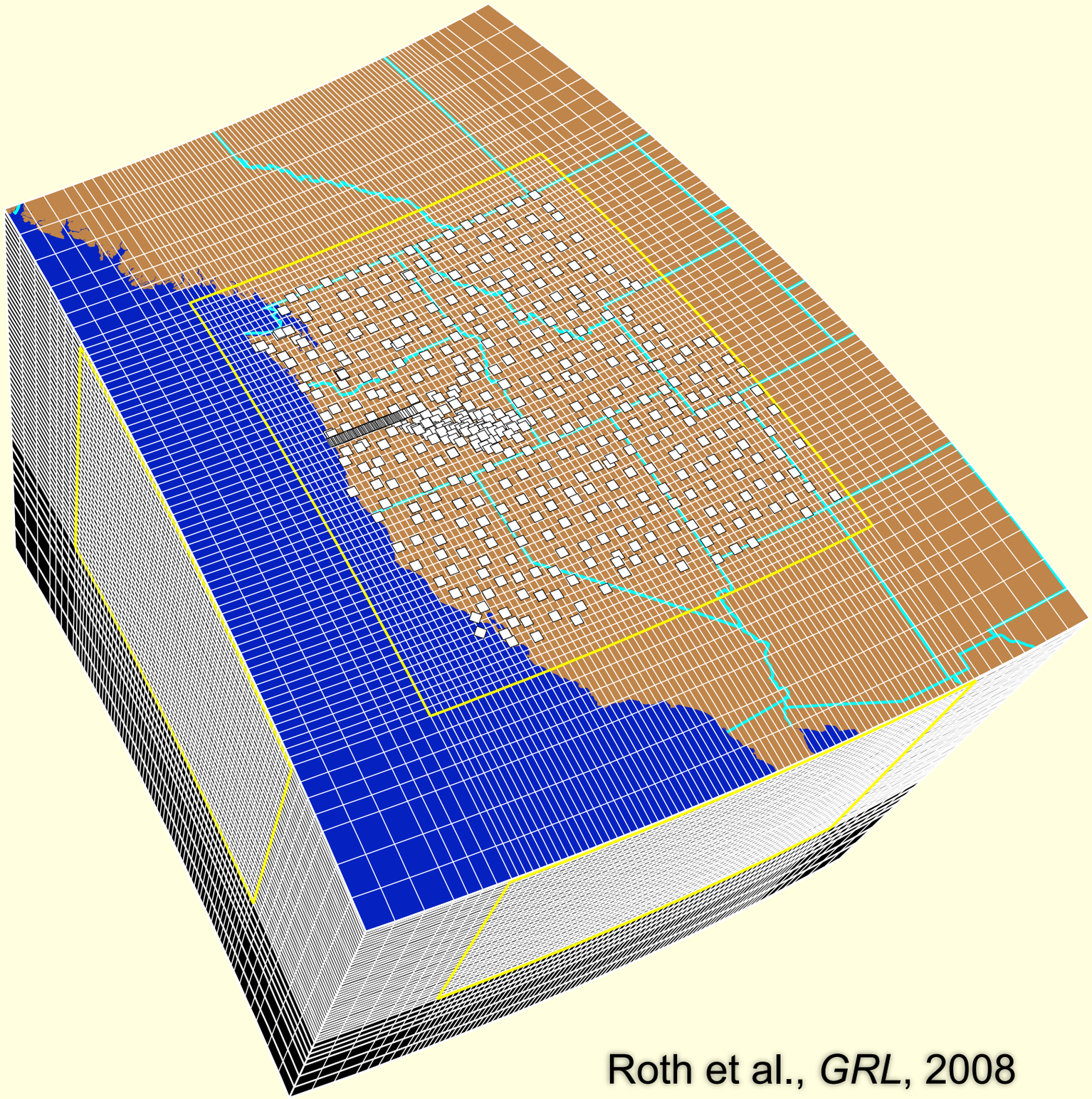
From http://www-lgit.obs.ujf-grenoble.fr/~virieuxj

Global Tomographic Images of the Africa "Superplume"

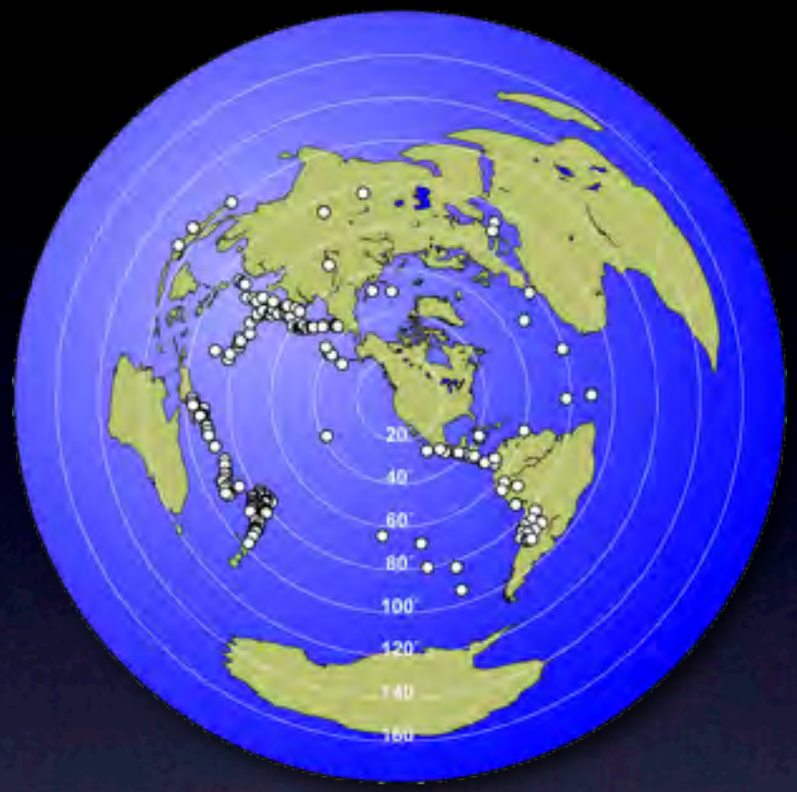


Ritsema et al. [1999]

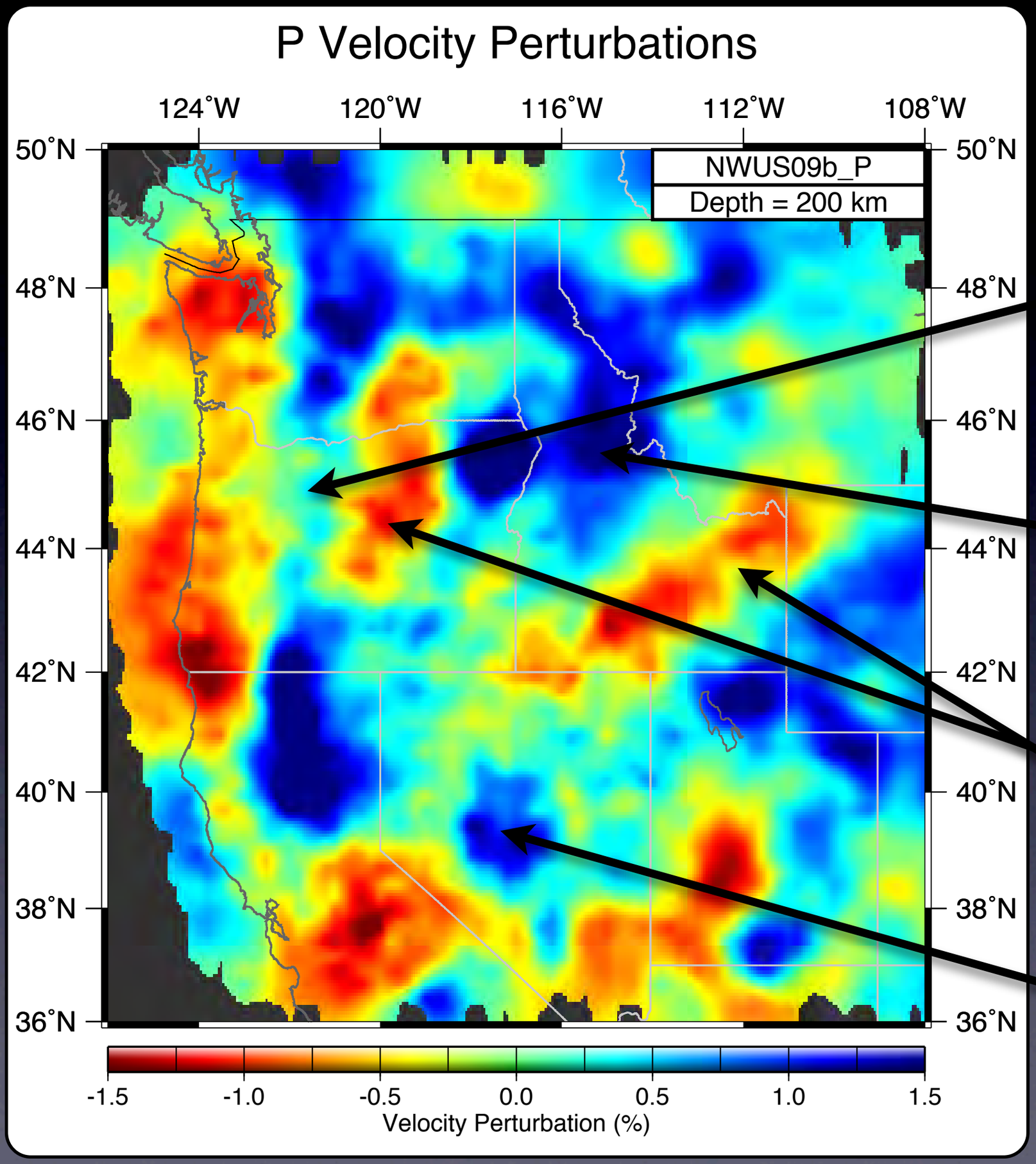
3-D Model: NW U.S. Tomography



Roth et al., *GRL*, 2008



- Inversion of relative delay times
- Ray-based method
- ~300,000 cells (knots)
- ~56,000 rays (P)
- ~58,000 rays (S)

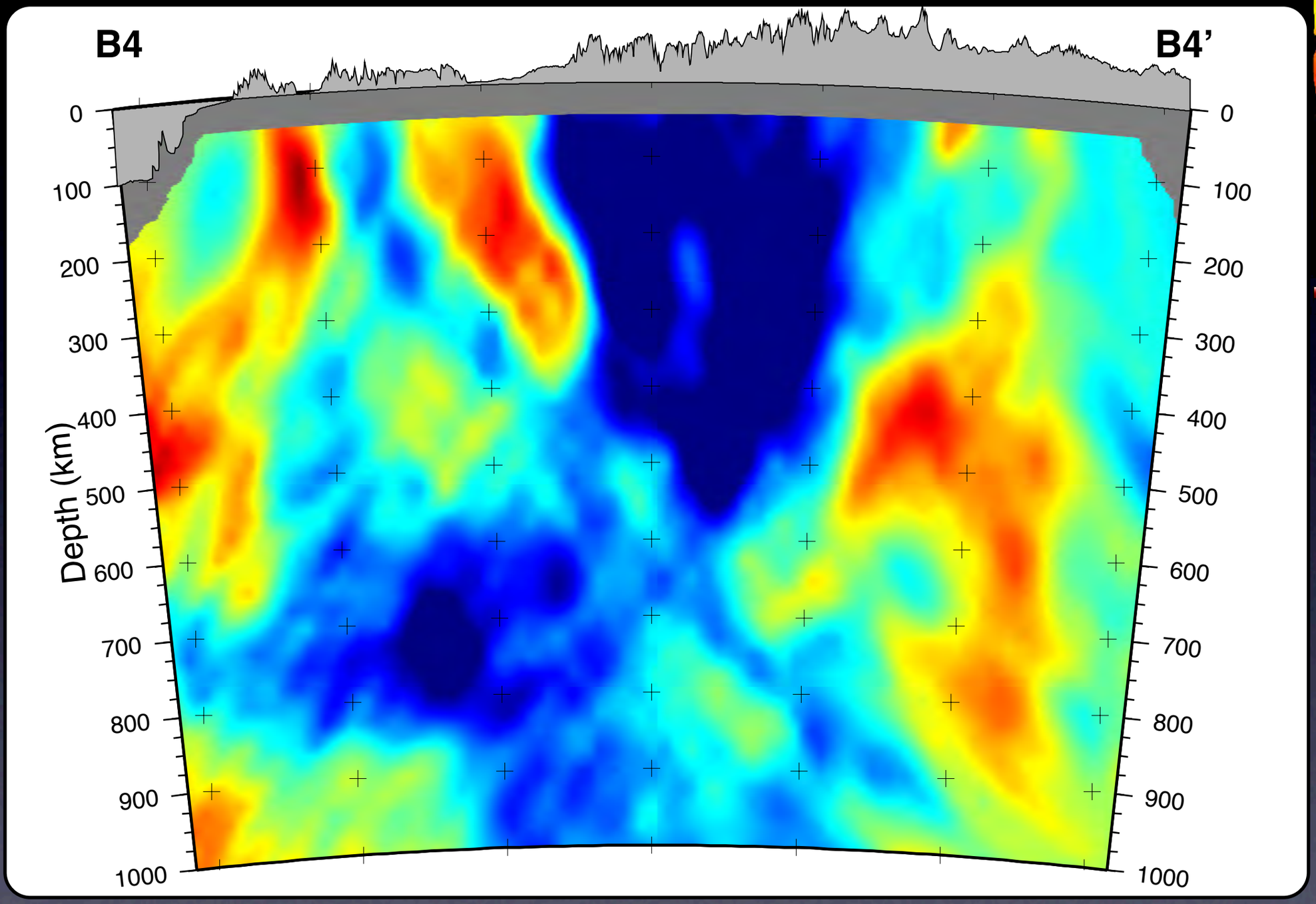


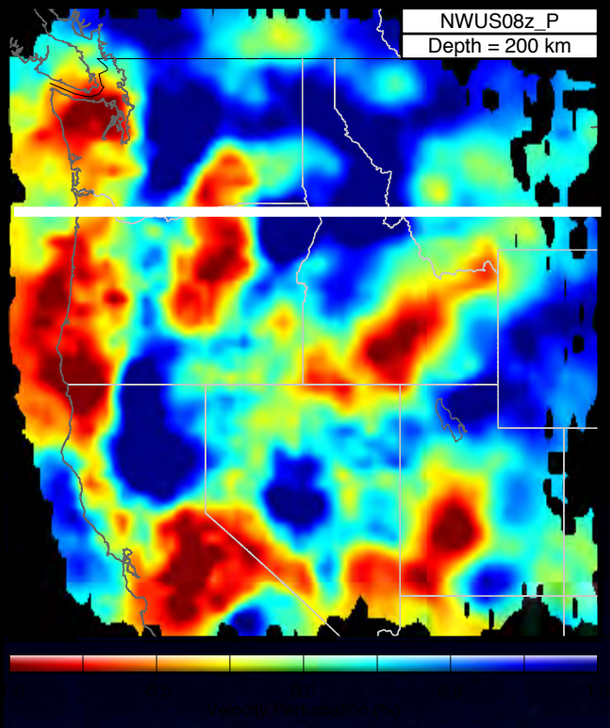
200 km depth slice

- *Apparent* gap in Juan de Fuca slab (inversion artifact)
- High velocities beneath cratonic North America
- Low velocities beneath central Oregon and SRP/Y
- Central Nevada anomaly

Roth et al., *GRL*, 2008

Cross Section: N. Oregon

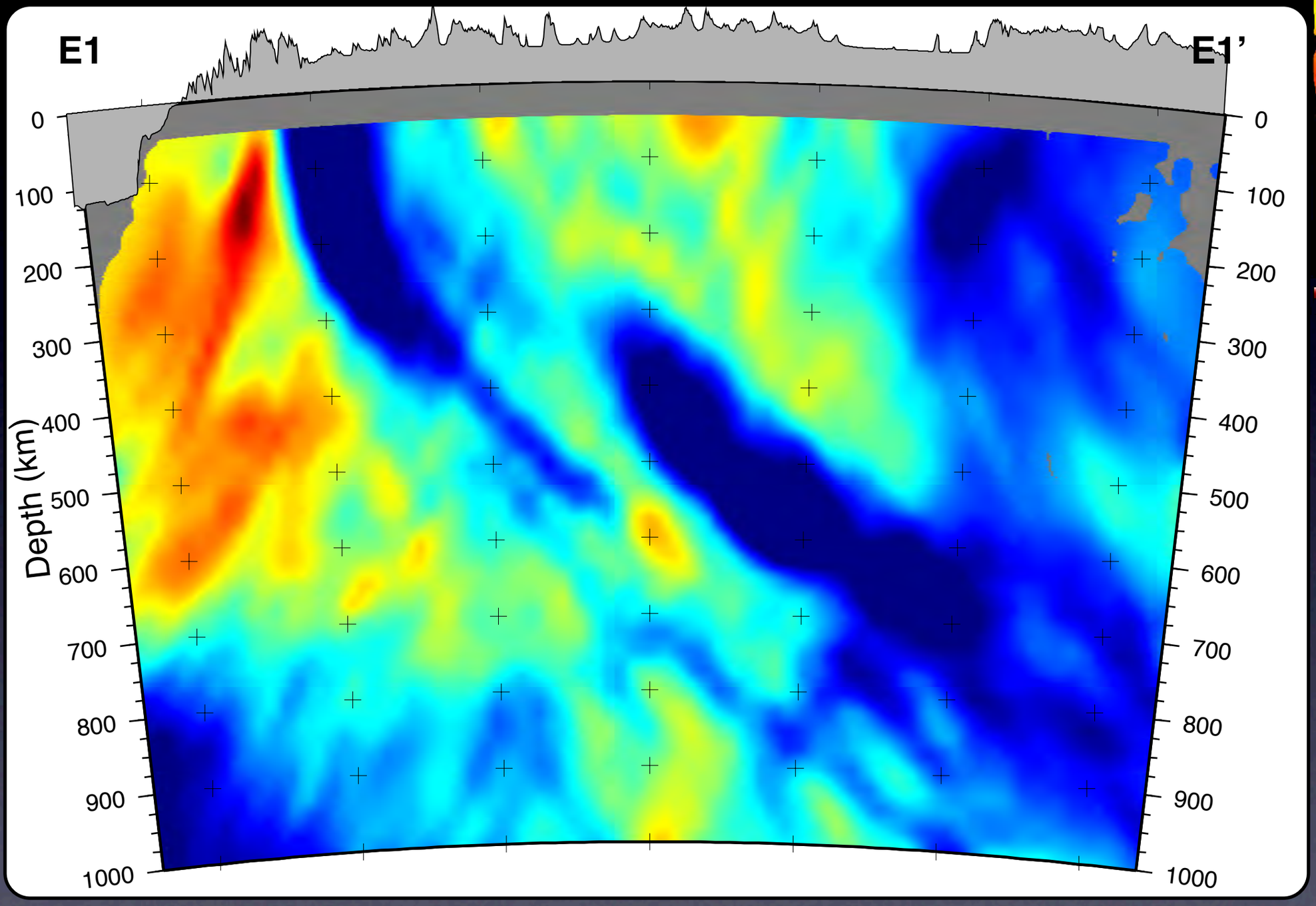


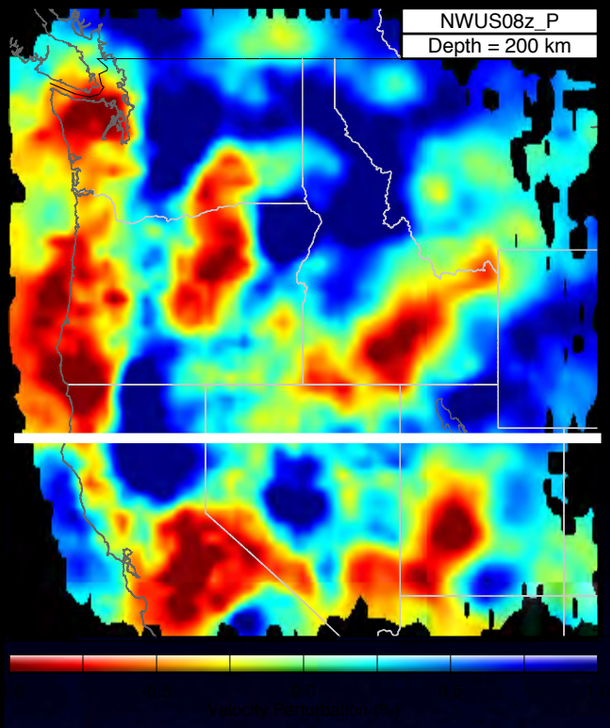


Roth et al., *GRL*, 2008

- *Apparent* missing Juan de Fuca slab (likely inversion artifact)
- Large zone of high velocities beneath Blue Mountains/Idaho Batholith

Cross Section: N. Nevada

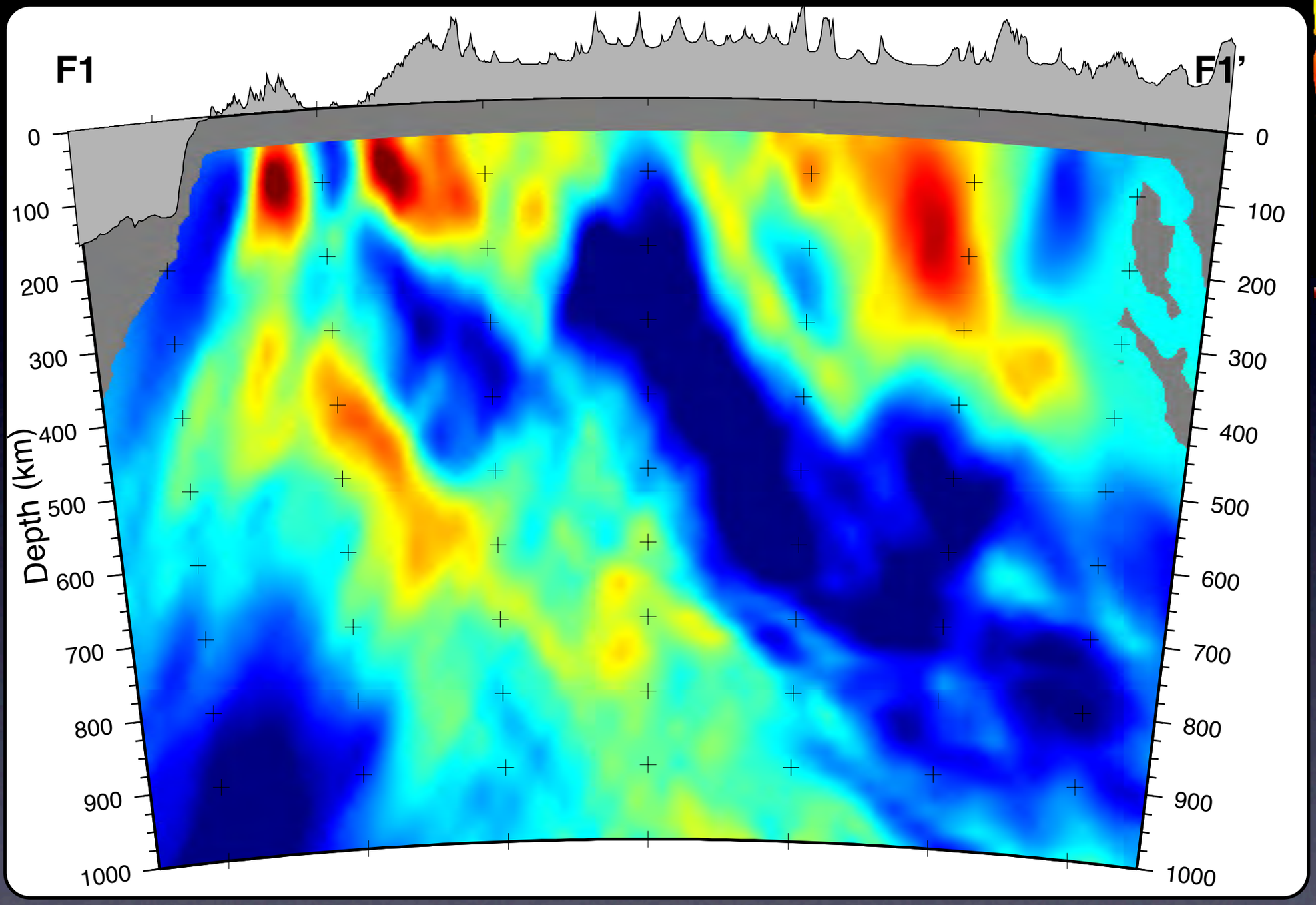


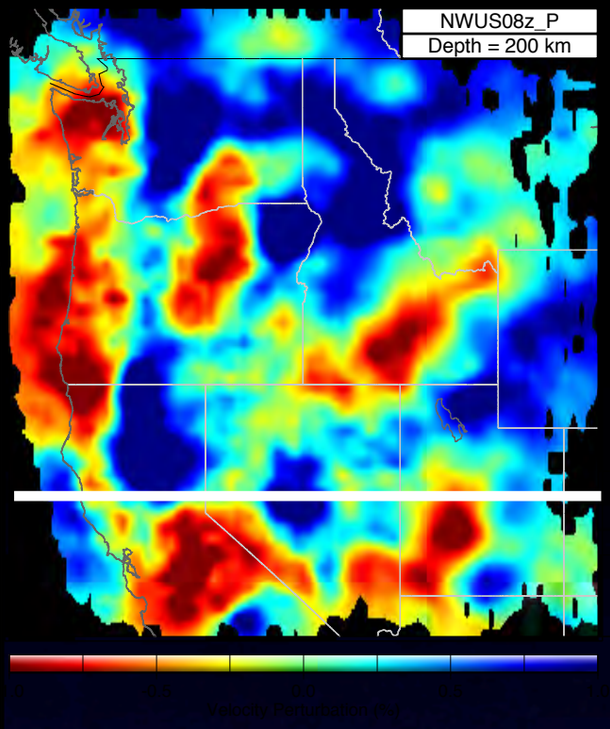


Roth et al., *GRL*, 2008

- Juan de Fuca slab extends to 500 km and perhaps deeper
- New discovery: Secondary slablike structure evident beneath Oregon/Idaho border

Cross Section: C. Nevada

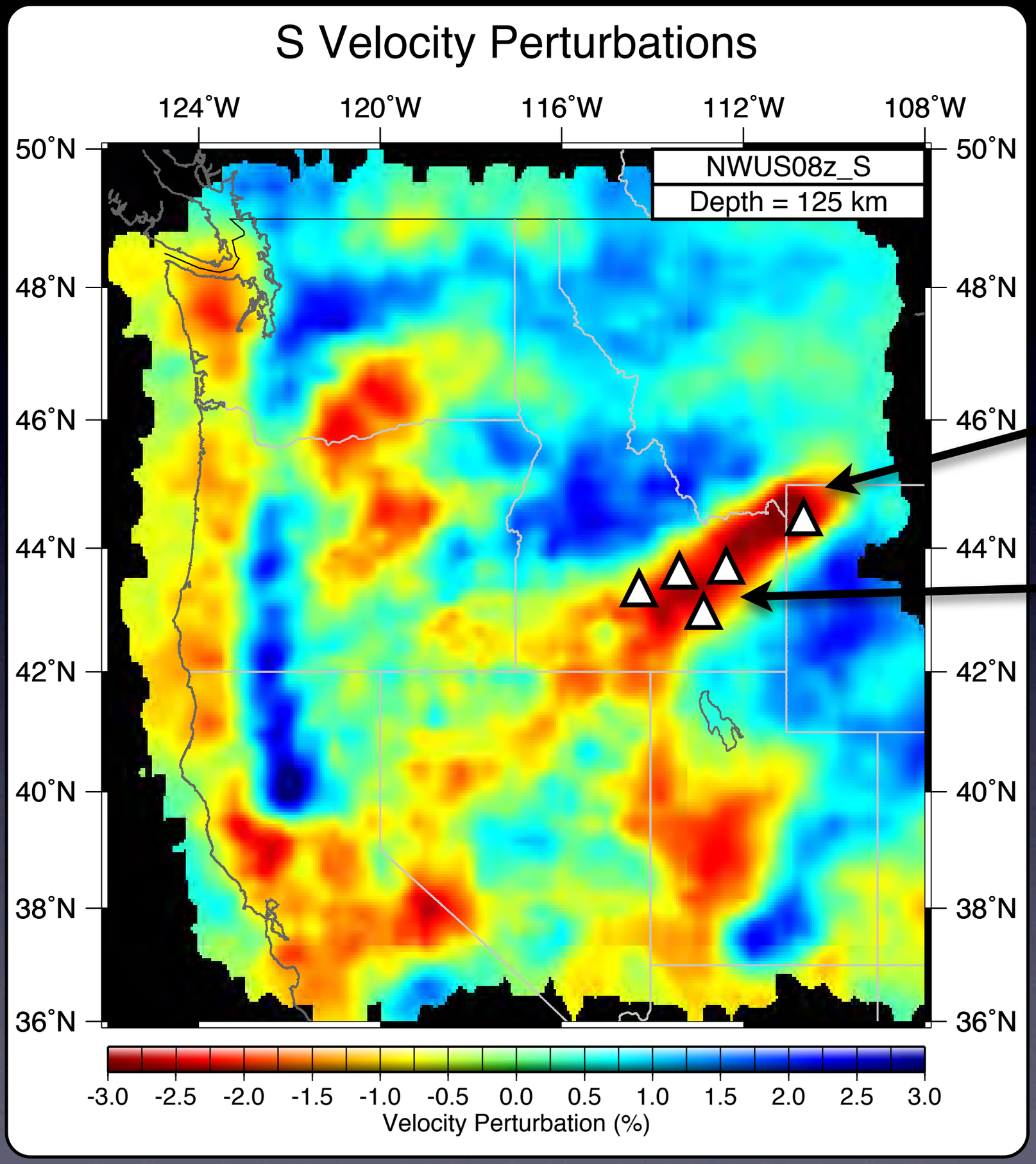




Roth et al., *GRL*, 2008

- Discovery: Large high velocity cylinder beneath the central Great Basin (interpreted by *West et al*., 2009)

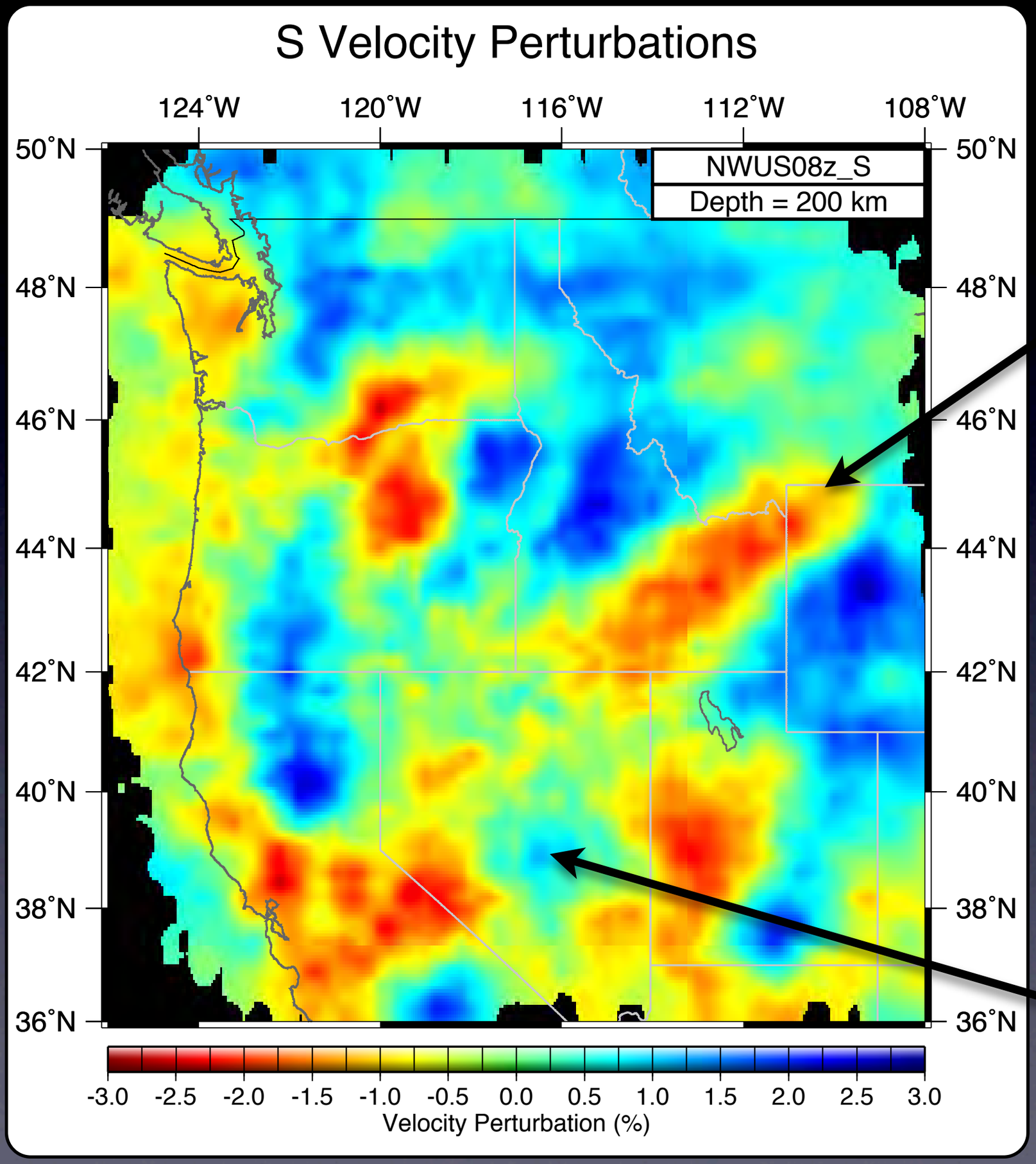
125 km depth slice



- Lowest S velocities beneath Yellowstone
- Low velocities persist through eastern Snake River Plain
- Colocated with Holocene volcanism

James et al., in prep, 2009

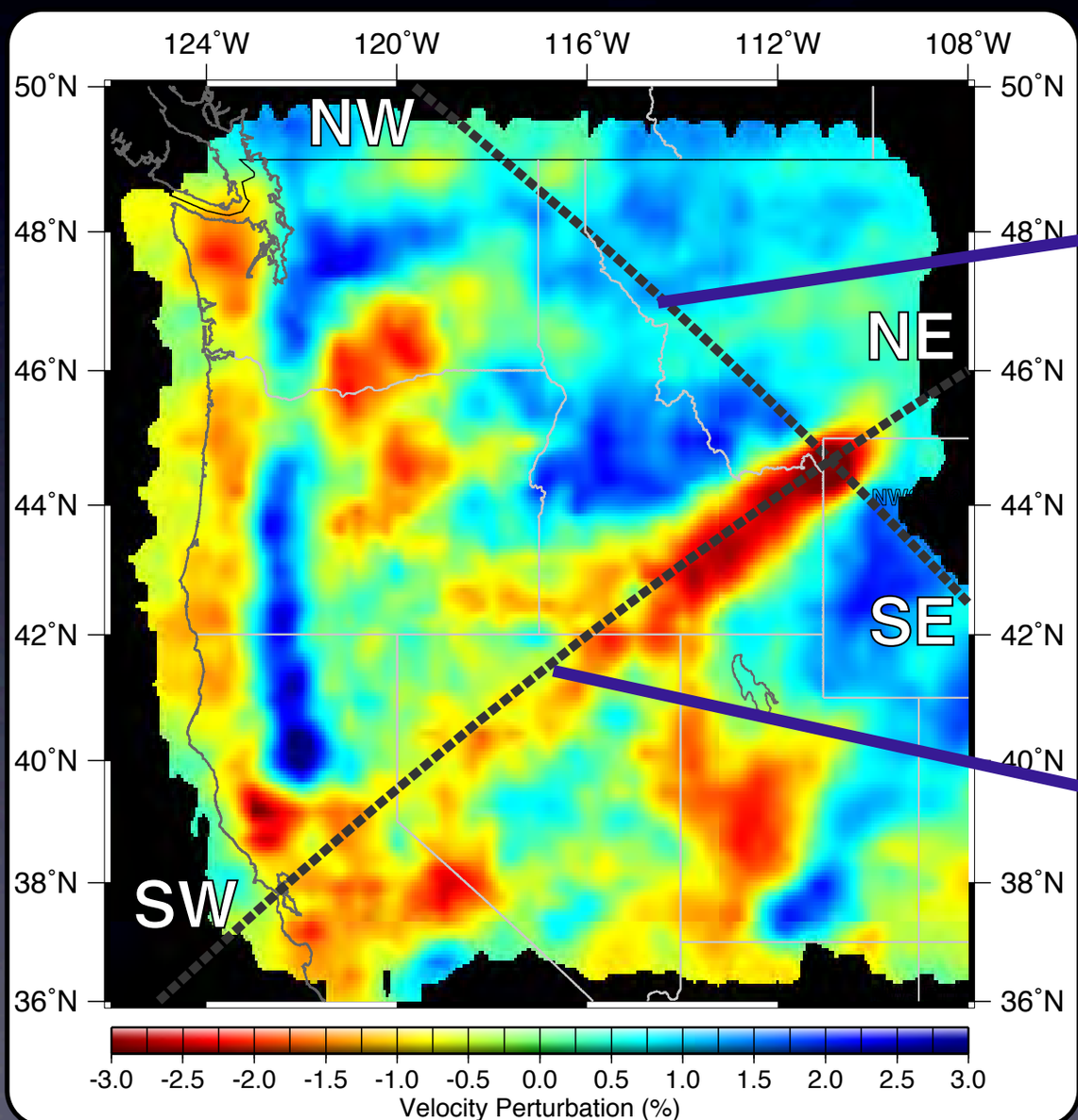
200 km depth slice

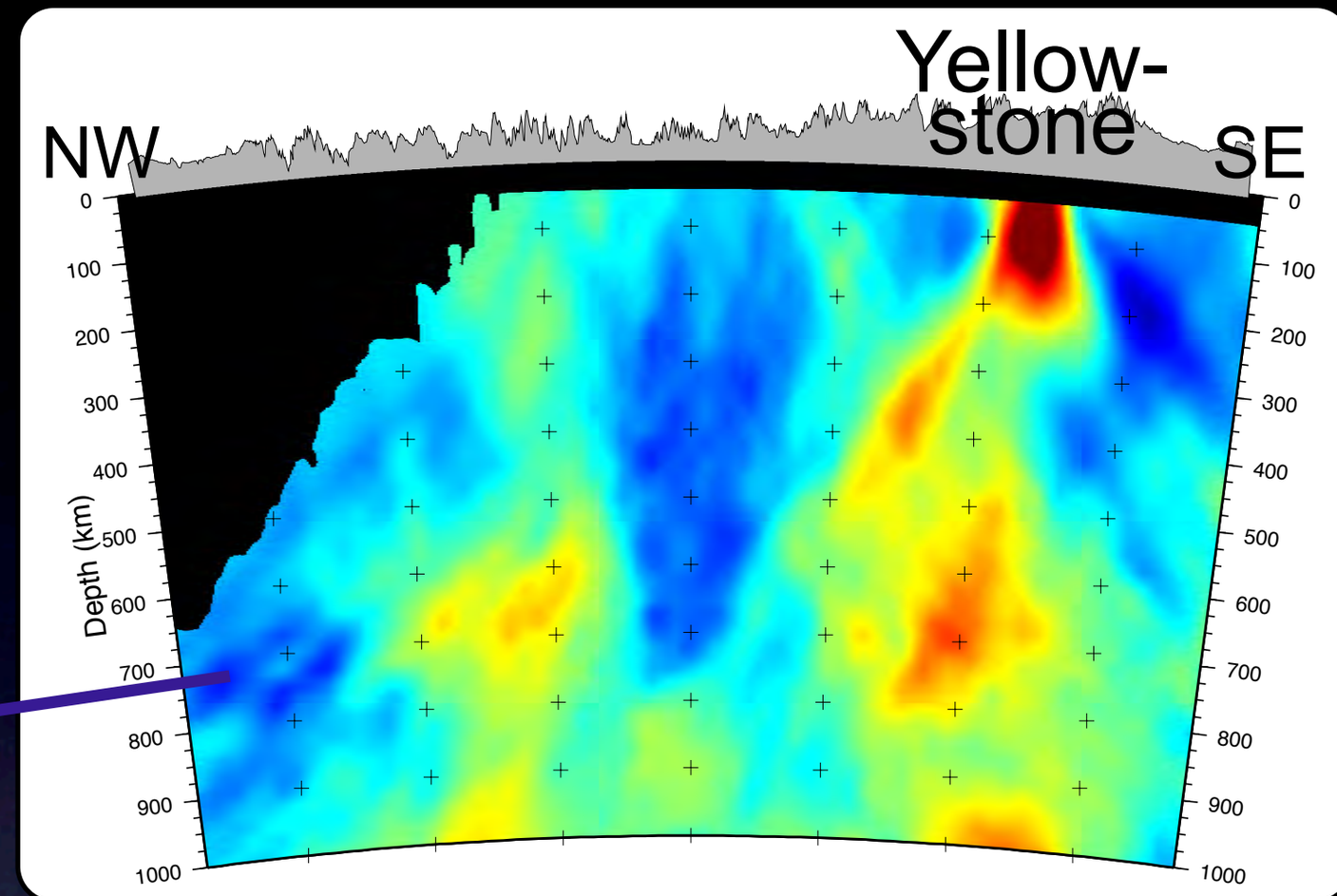


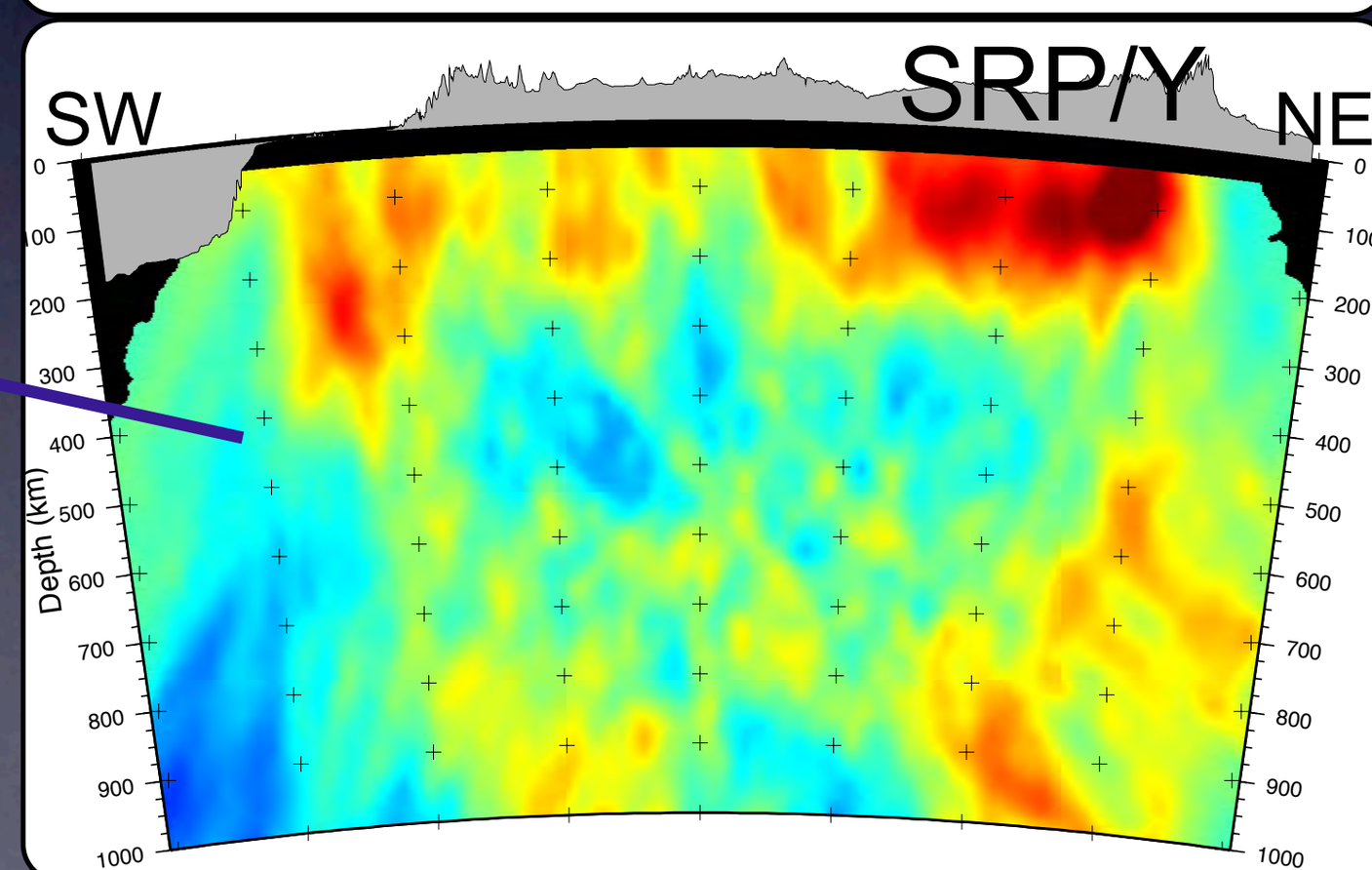
- Yellowstone and Snake River Plain have identical (weak) signature at 200 km and deeper
- Weaker signature than Juan de Fuca wedge and Great Basin / Rio Grande Rift asthenosphere
- Central Nevada anomaly evident

James et al., in prep, 2009

No Simple Mantle Plume Beneath Yellowstone

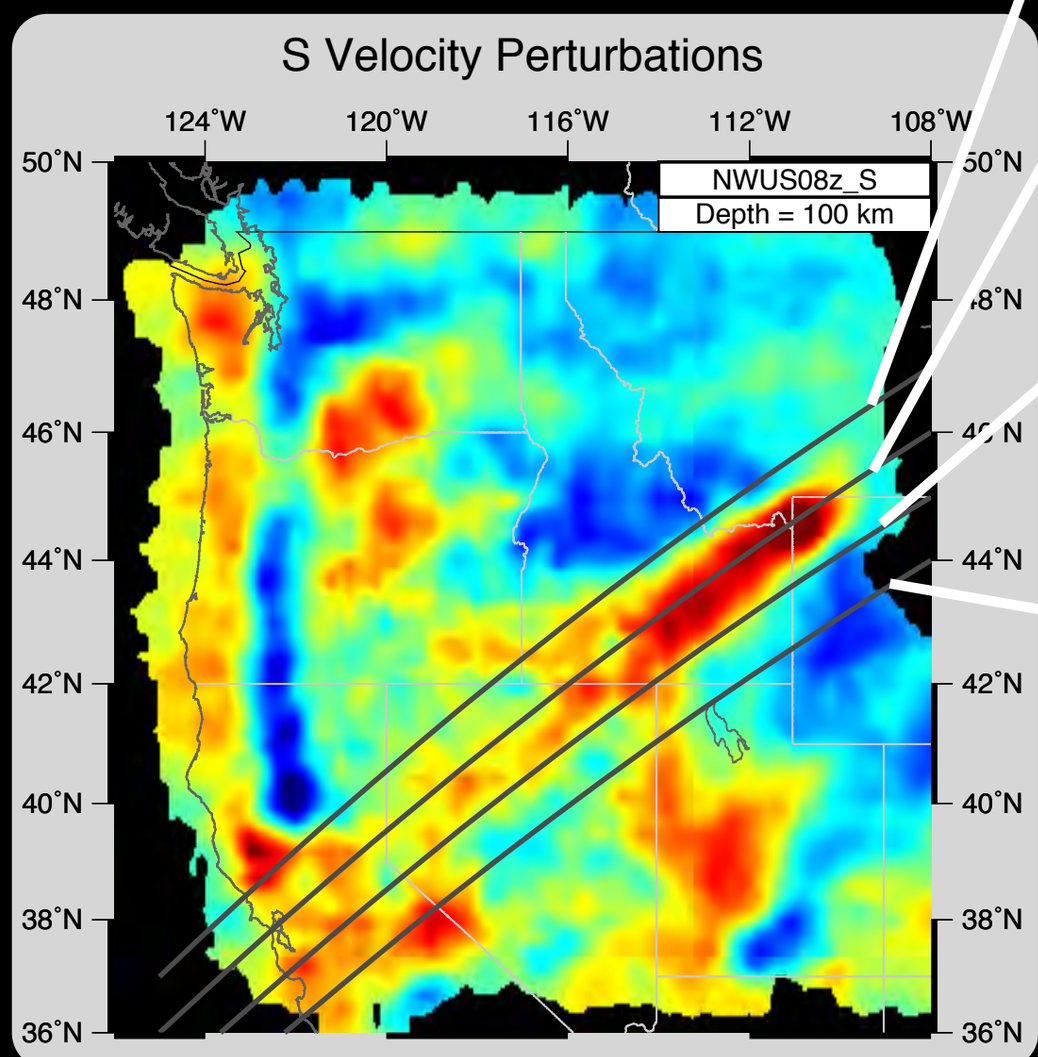


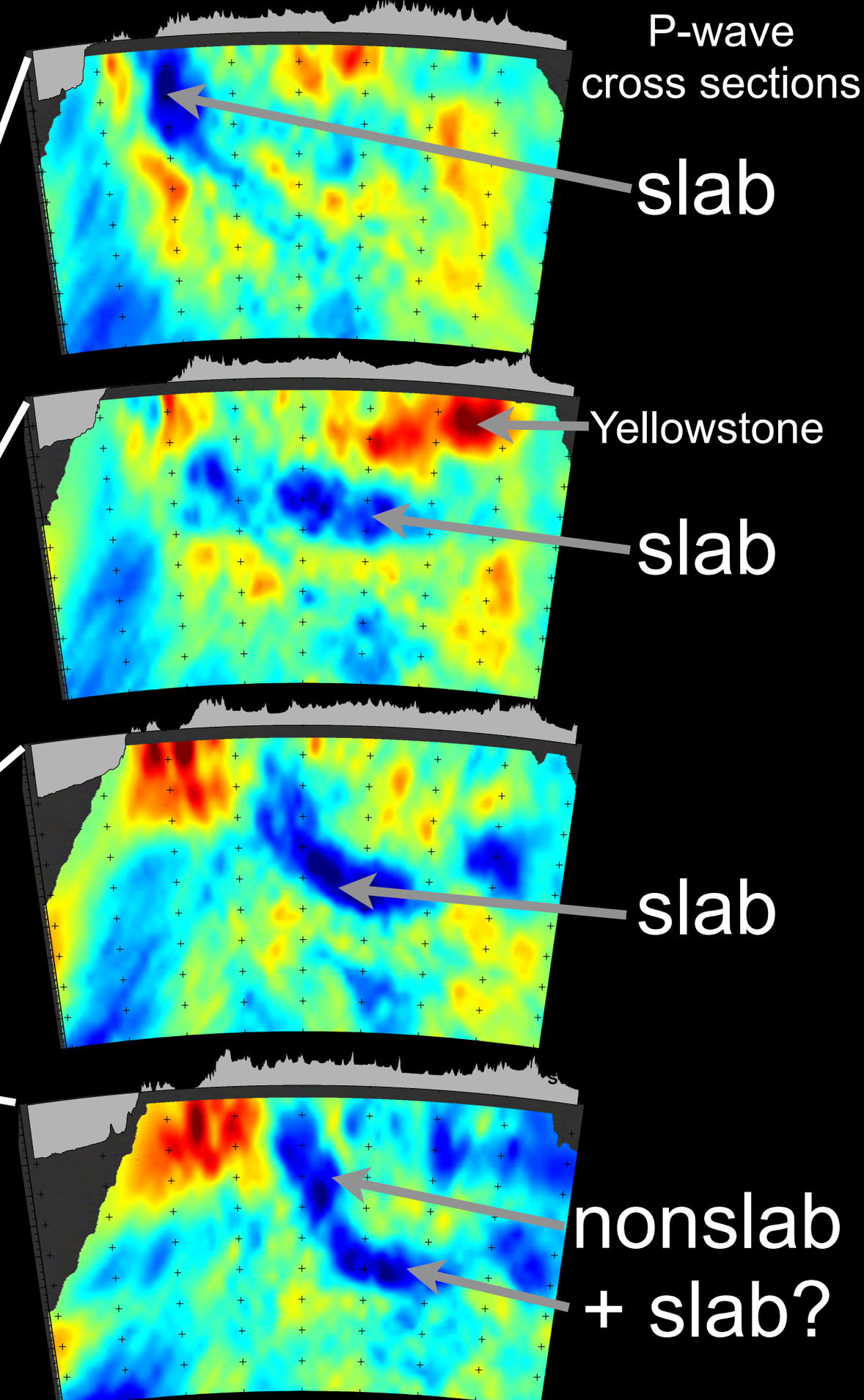




James et al., in prep, 2009

Complex Slab Morphology Beneath Snake River Plain / Yellowstone System

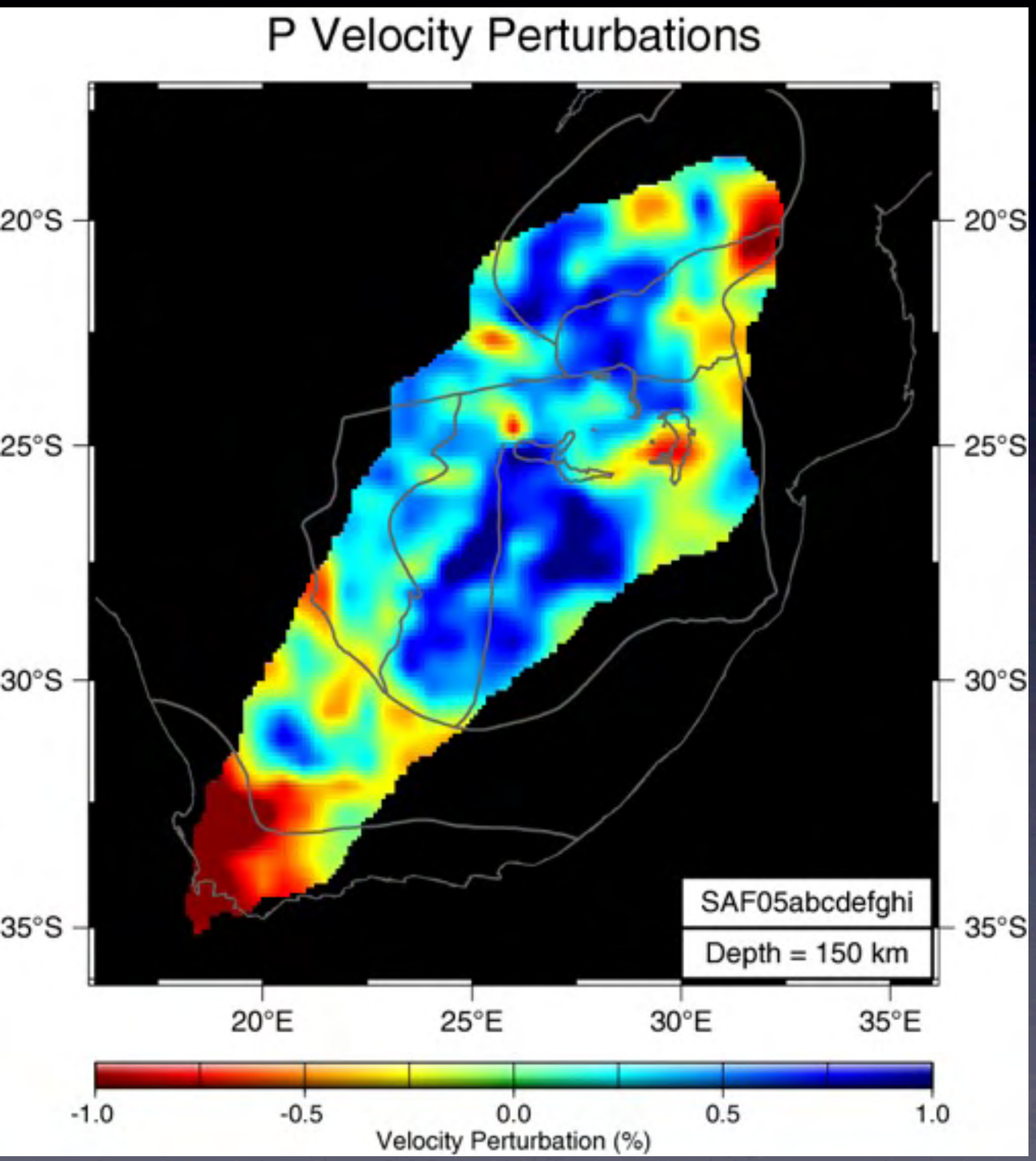


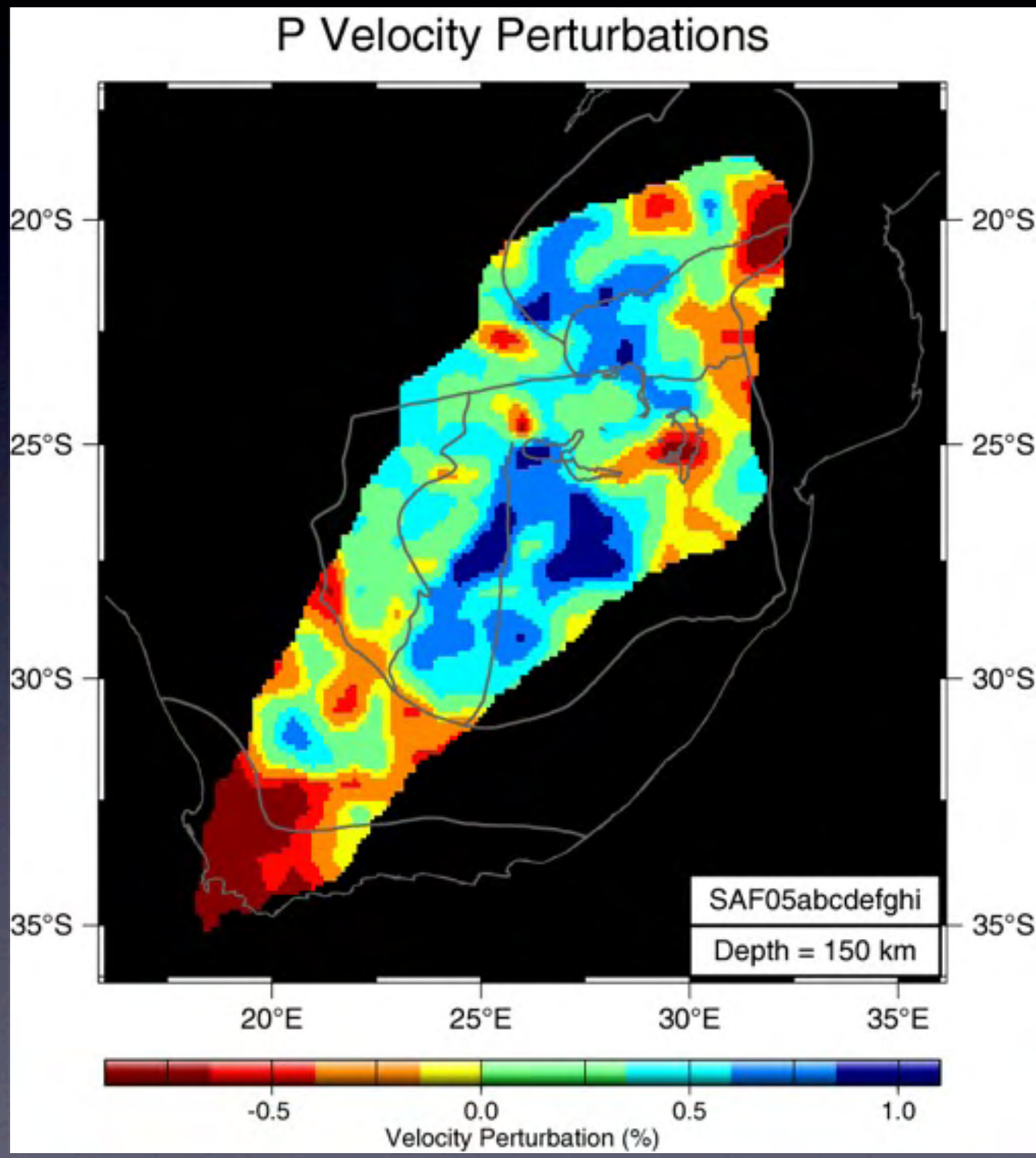


Body Wave Tomography: Weaknesses / Caveats

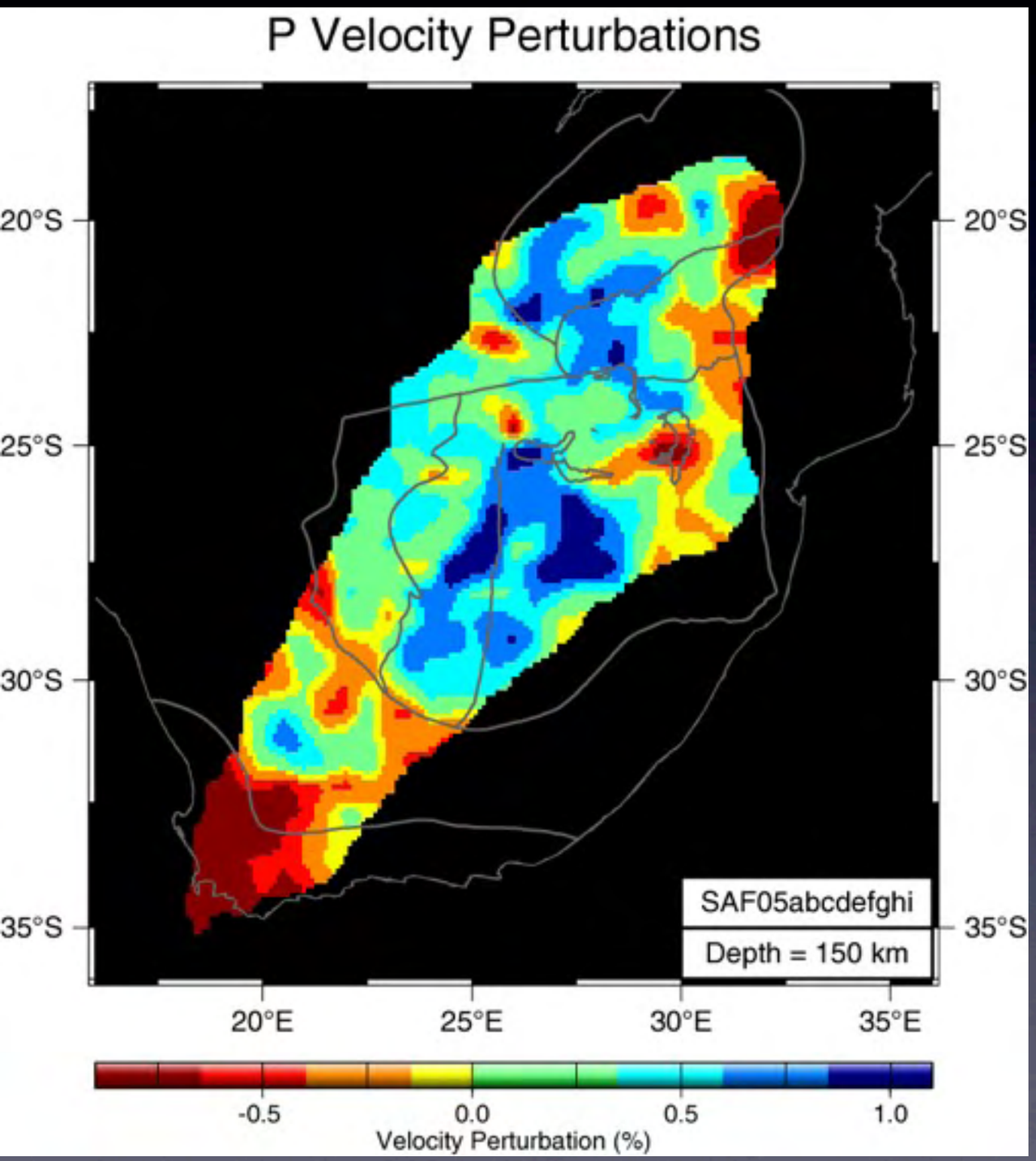
- Resolution: Model will always be a blurry (smoothed) image of real structure
- Regularization: How much bias does model have from damping and smoothing?
- Misfit: How well does model fit data? Is it unique?
- Many methods: What happens when they don't agree?
- Be cognizant of:
  - Very small features: avoid interpretation!
  - Color schemes and isocontours
  - Streaks pointing in direction of earthquakes used in inversion

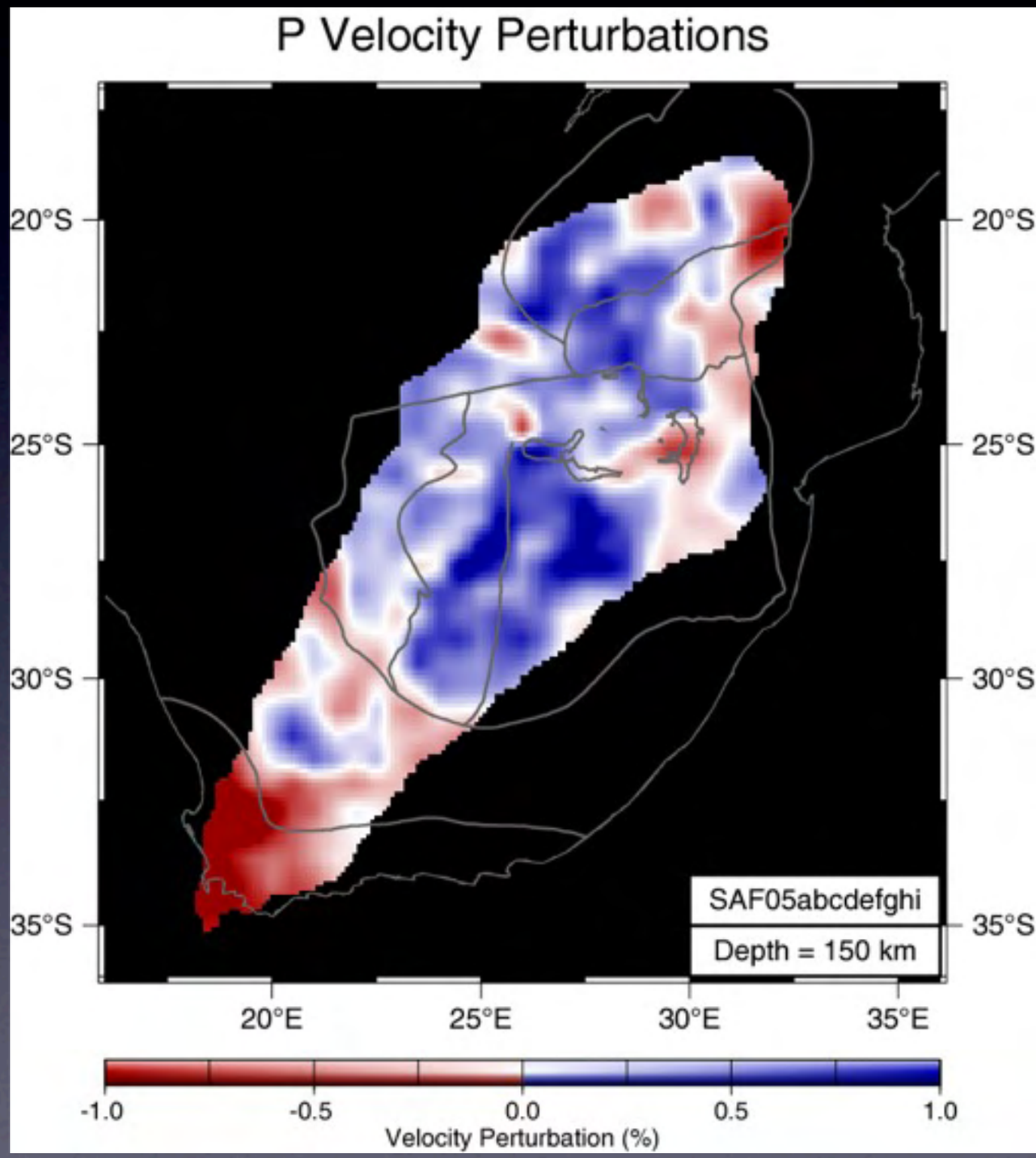
Same Model, Different Colors



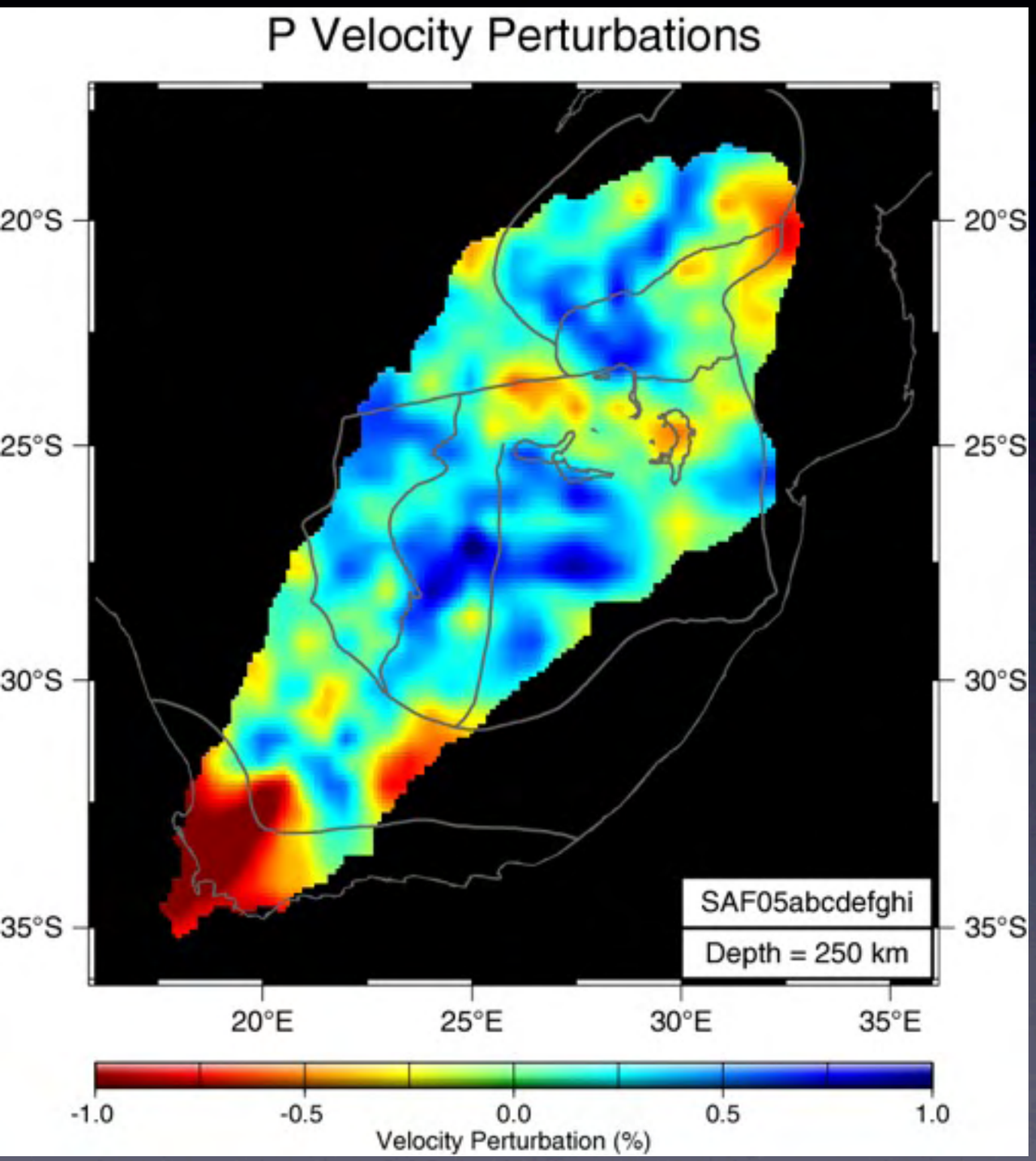


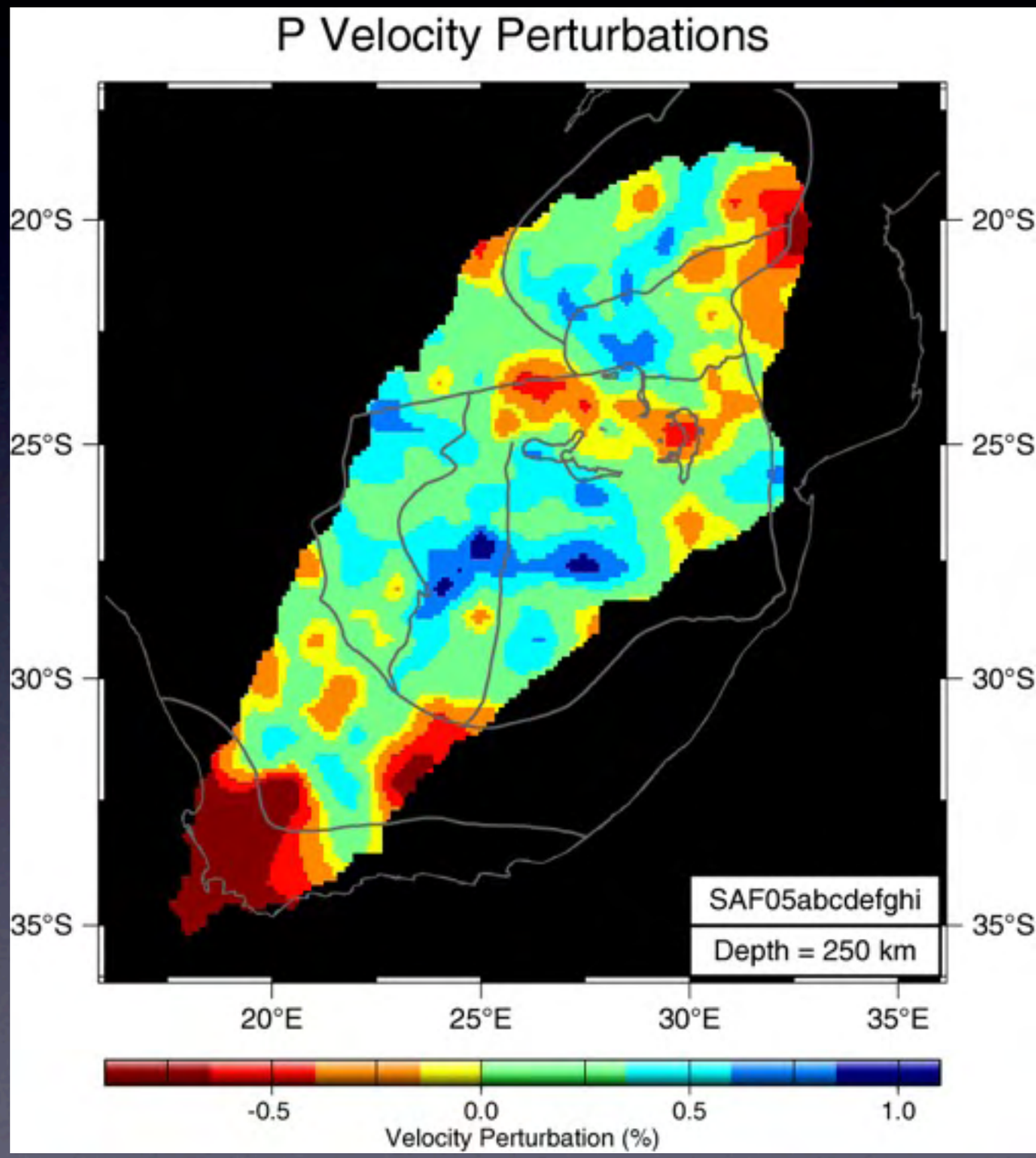
Same Model, Different Colors



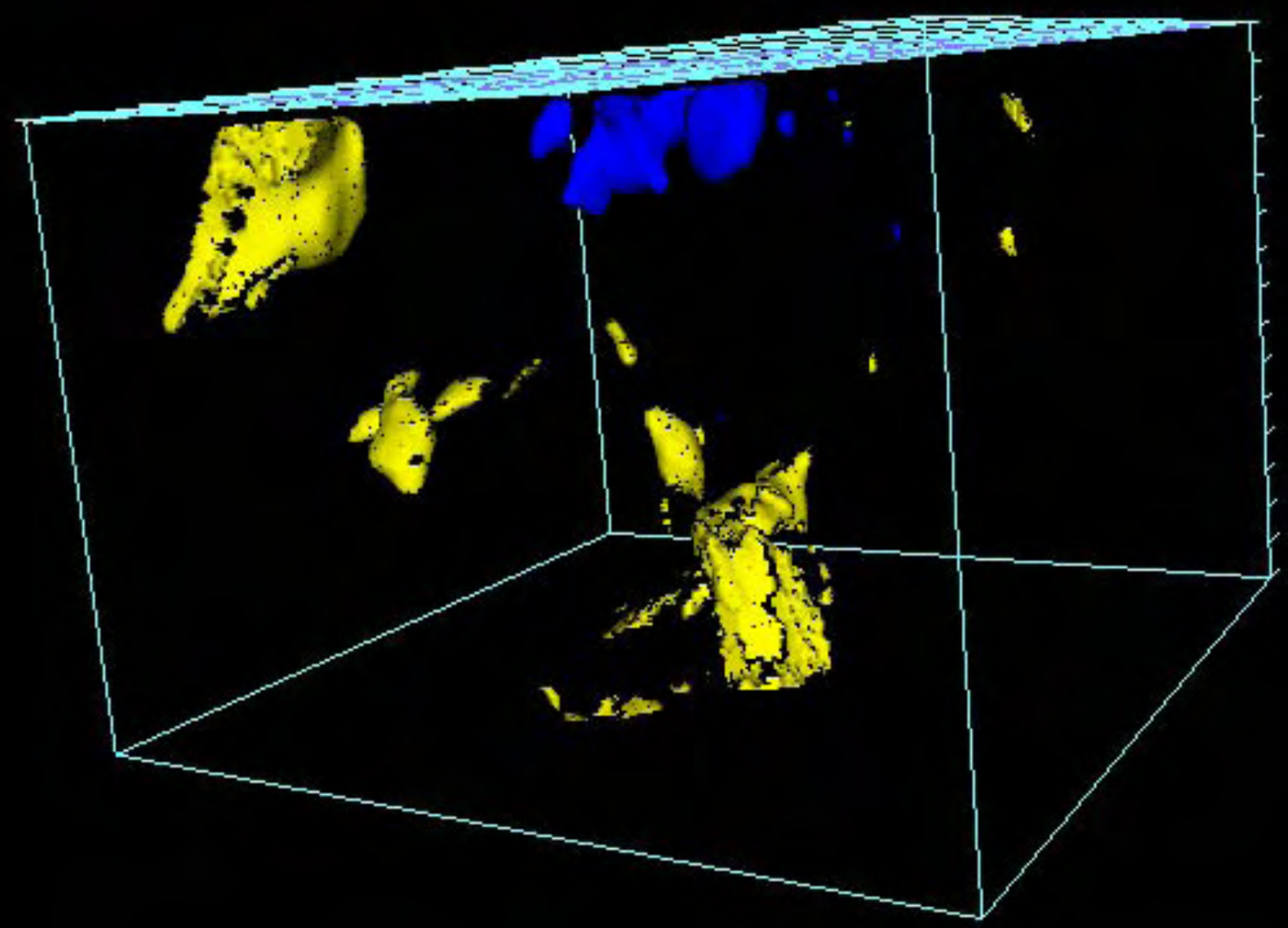


Same Model, Different Colors

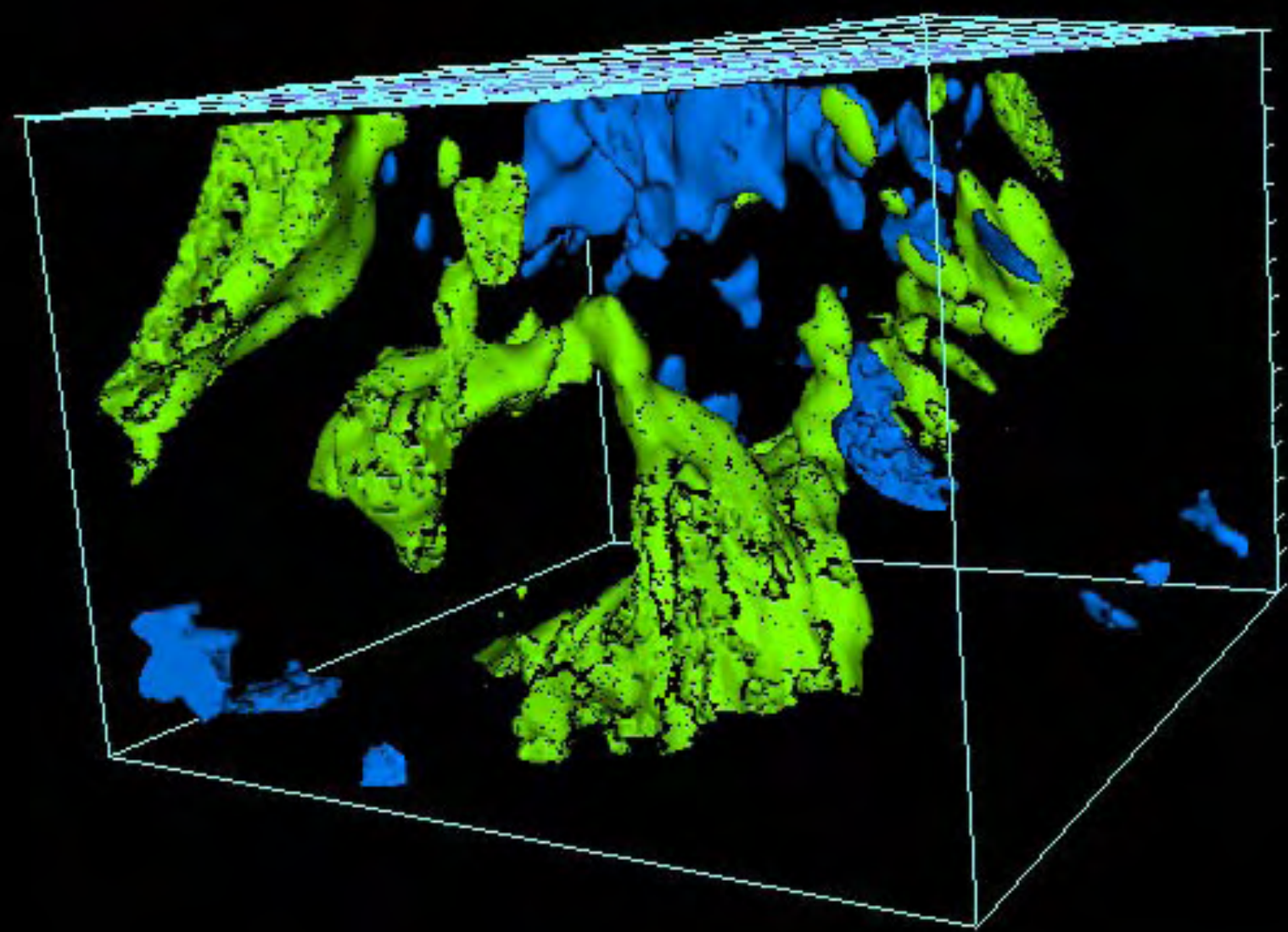




Isocontours: +/-0.6%



Isocontours: +/-0.4%



Body Wave Tomography: Strengths

- Many methods: great when they agree!
- Results can be easy to interpret visually
- Dense arrays provide higher level of resolution
- Surface wave inversions can provide absolute velocities
- Complementary observations:
  - Receiver functions
  - Seismic anisotropy
  - Combined surface wave/body wave inversions